DOI: 10.24996/ijs.2025.66.4.23

ISSN: 0067-2904

The Influence of Fear on the Dynamics of Harvested Prey-Predator Model with Intra-Specific Competition

Hiba Abdullah Ibrahim*, Rehab Noori Shalan, Dina Aljaf
Department of Mathematics, College of Science, University of Baghdad, Baghdad, Iraq

Received: 23/6/2023 Accepted: 8/5/2024 Published: 30/4/2025

Abstract
The influence of fear on the dynamics of harvested prey-predator model with intra-specific competition is suggested and studied, where the fear effect from the predation causes decreases of growth rate of prey. We suppose that the predator attacks the prey under the Holling type IV functional response. he existence of the solution is investigated and the bounded-ness of the solution is studied too. In addition, the dynamical behavior of the system is established locally and globally. Furthermore, the persistence conditions are investigated. Finally, numerical analysis of the system is carried out.

Keywords: Fear effect, Holling type IV functional response, Intra-specific competition, Stability analysis, Persistence.

تأثير الخوف على ديناميكيات نموذج الفريسة المفترسة المحصود مع التنافس الضمني

هبة عبدالله ابراهيم*, رحاب نوري شعلان, دينا الجاف
قسم الرياضيات , كلية العلوم, جامعة بغداد, بغداد, العراق

الخلاصة
تم اقتراح ودراسة تأثير الخوف على ديناميكيات نموذج الفريسة المفترسة المحصود مع التنافس الضمني، حيث يؤدي تأثير الخوف من الافتراس إلى انخفاض معدل نمو الفريسة. نفترض أن المفترس يتغذى على الفريسة وفقًا لدالة الاستجابة هولنك من النوع الرابع. يتم التحقيق في وجود الحل وحدوده. يتم إنشاء السلوك الديناميكي للنظام محليًا وشاملا. يتم التحقيق في ظروف الثبات. أخيرًا ، تم إجراء التحليل العددي للنظام.

1. Introduction

The prey-predator relation is one of the most important tools in the environment system. as a result of wide incidence and significance, the interaction between predators and their prey have been commonly studied. This subject will continue and stay to be one of the essential subjects in both ecology and biology, see [1-2]. In recent decades, mathematical modeling has a large effect as a tool for understanding the processes of biological. In the real world, all the species is interacting with each other in various ways. The interactions between species take different forms, for example, mutualism, prey-predator, and competition for food, etc.

*Email: hiba.ibrahim@sc.uobaghdad.edu.iq

The first major attempt to investigate the evolution of predators and prey populations is due to Lotka, 1925 and independently by Volterra, 1926 which called Lotka-Volterra model [3].

Several kinds of prey-predator models with various biological factors were studied by many investigators, such as fear, harvesting, refuge, stage structure, cannibalism and etc., see [4–8]. Fakhry and Naji [4] studied the effect of fear on the prey-predator model, they found that rising the rate of fear causes a decreasing in the predators and the system stay persists with the co-existence equilibrium point. Haque and Sarwardi [7] proposed a prey-predator model with prey refuge and harvesting, they reached to the effect of refuge plays a significant role in regulating the dynamics of the system. Also, the functional response of species is an important in the ecological systems. Many researchers studied prey-predator models with different types of functional response [9-13]. Naji and Shalan [10] presented a prey-predator model with Holling type IV functional response and intra-specific competition, they found the growth rate and intra-specific competition parameters had a stabilizing effect on the system.

In ecological systems, the influence of predator on the prey perhaps direct, indirect or together. In the direct influence, the predator kills the prey directly [14]. Whilst, in the indirect influence, the predator causes fear to the prey and thus leads to a decrease in the growth rate of the prey [15]. Recent works displayed the impact of the fear to the ecological systems [16-19]. Tian and Li [18] presented a model of prey -predator involving fear and harvesting, they found when the fear is small, the coexistence equilibrium point is unsteady and a limit cycle presents. They also observed that as the effect of fear increases then the coexistence equilibrium point be stable while the limit cycle vanishes.

On the other hand, many investigators focused on the study of harvesting of the species on the dynamics of ecological system, see [20-24]. Raymond et.al [21] suggested a prey-predator fishery model with harvesting, they deduced that if the harvesting rate exceeds the growth rate, then the population would be extinct with time. Several types of harvesting have been proposed and studied such as constant harvesting, nonlinear and proportional harvesting see [25-30].

The objective of this paper is to understand how the biological factors as fear and harvesting may effect on the dynamic of the model.

In this paper, the influence of fear on the dynamics of harvested prey-predator with intra-specific competition is suggested and studied. We presume that the effects of fear and harvesting are included on the model and the predator attacks the prey according to Holling type IV functional response. In the next section, the model is mathematically formulated and the bounded-ness of the proposed system is studied. In section three, the equilibrium points analysis and local stability of the system are investigated, while the conditions of persistence are determined in section four. after that, Numerical analysis is carried out to explain the analytical outcomes in section five. Finally, conclusions are included in section six.

2. Model Formulation

The influence of fear on the dynamics of harvested prey-predator model with intra-specific competition is suggested and studied. Consider that the prey's density $x(t)$ and the predator's density $z(t)$ at time t. Suppose that the prey logistically grows in the absence of a predator with a growth rate $r > 0$ and carrying capacity $k > 0$. However, the fear effect from the predation causes decreases in the growth rate with a constant fear rate $\theta > 0$. The predator eats the prey under the Holling type IV functional response with maximum attack rate $a > 0$, the direct measurement of the predator immunology from the prey $\beta > 0$ and the half saturation level $b > 0$. If the prey is unavailable, then the predator is eliminated with death rate $d > 0$. The predator competes with its species with intra-specific competition rate $\alpha > 0$. Moreover, the positive constants e, E_1 and E_2 represent the conversion rate, harvesting efforts for the prey and predator, respectively. Now the dynamics of the harvested prey-predator model with fear and intra-specific competition is represented by the following set of differential equations:

$$\begin{aligned} \frac{dx}{dt} &= \left(\frac{rx}{1+\theta z}\right)\left(1-\frac{x}{k}\right) - \frac{a\beta xz}{x^2+\beta x+\beta b} - E_1 x, \\ \frac{dz}{dt} &= \frac{ea\beta xz}{x^2+\beta x+\beta b} - dz - \alpha z^2 - E_2 z. \end{aligned} \tag{1}$$

Obviously, the domain of system (1) is given by $R_+^2 = \{(x,z) \in R^2 / x \geq 0, z \geq 0\}$. Moreover, the right hands of the system (1) are in the C^1 which are Lipschitzain functions. Therefore, the solution to the system (1) exists uniquely. We notice that an ecological model is well post if and only if it is a bounded model. So the next theorem gives the solution to the system (1) is uniformly bounded. Note that, the survival condition of coexistence of each species in system (1) is given by

$$r > E_1. \tag{2}$$

Therefore, from now onward, we presume that condition (2) always holds.

Theorem (1): All the solutions to system (1) are uniformly bounded.

Proof: According to 1st equation, its noted that

$$\frac{dx}{dt} \leq rx\left(1-\frac{x}{k}\right) - E_1 x \; . \tag{3}$$

Then, by direct computation and condition (2), we obtain the following

$x(t) \leq \frac{k(r-E_1)}{r}, \;\; \forall t \geq 0.$

Let $V(t) = ex(t) + z(t)$, then

$\frac{dV}{dt} \leq \frac{er}{1+\theta z}x - LV$, where $L = min\{E_1, d + E_2\}$, and it gives

$\frac{dV}{dt} + LV \leq M$, where $M = \frac{ek(r-E_1)}{1+\theta z}$. By means of the Granwall lemma, we get

$V(t) \leq \frac{M}{L}(1 - e^{-Lt}) + V_0 e^{-Lt}$. For $t \to \infty$, we have

$V(t) \leq \frac{M}{L}$. Therefore, system (1) are uniformly bounded.

3. Equilibrium Analysis and Local Stability

System (1) has three positive equilibrium points, namely, $y_i = (x_i, y_i), i = 0,1,2.$

1. The equilibrium point $y_0 = (0,0)$ always exists.

2. The equilibrium point $y_1 = (\hat{x}, 0)$, where $\hat{x} = \frac{k(r-E_1)}{r}$, always exists under the survival condition (2).

3. The interior equilibrium point $y_2 = (x^*, z^*)$ exists, where

$$z^* = \frac{1}{\alpha}\left[\frac{ea\beta x^* - (d+E_2)\left(x^{*2}+\beta x^*+\beta b\right)}{\left(x^{*2}+\beta x^*+\beta b\right)}\right]. \tag{4}$$

and x^* is a positive root of the polynomial of order seven.

$$B_1 x^7 + B_2 x^6 + B_3 x^5 + B_4 x^4 + B_5 x^3 + B_6 x^2 + B_7 x + B_8 = 0. \tag{5}$$

where $B_1 = -\frac{\alpha r}{k} < 0$,

$B_2 = \alpha r\left(1 - \frac{3\beta}{k}\right) + E_1(\theta(d + E_2) - \alpha)$,

$B_3 = 3\left[\alpha r\beta\left(1 - \left(\frac{b-\beta}{k}\right)\right) + E_1\beta(\theta(d + E_2) - \alpha)\right] - E_1 ea\theta\beta$,

$B_4 = 3\left[\alpha r\beta\left(b+\beta-2\frac{\beta b}{k}\right)+E_1\beta b(\theta(d+E_2)-\alpha)(b+\beta)\right]-\frac{\alpha r\beta^3}{k}+\ \alpha\beta(d+E_2)\big(1-\theta(d+E_2)\big)-2E_1 ea\theta\beta^2,$

$$B_5 = 6\alpha r\beta^2 b+2\alpha r\beta^3-3\frac{\alpha r\beta^2 b^2}{k}-3\frac{\alpha r\beta^3 b}{k}-a^2\beta^2 e+2\frac{a^2\beta^2 e\theta(d+E_2)}{\alpha} + 2a\beta^2(d+E_2)-\frac{a\theta\beta^2(d+E_2)^2}{\alpha}+6E_1\theta\beta^2 b(d+E_2)-4E_1\alpha\beta^2 b - 2E_1\theta ea\beta^2 b-E_1\alpha\beta^3-E_1\theta ea\beta^3+E_1\theta\beta^3(d+E_2),$$

$$B_6 = 3\alpha r\beta^2 b\left(b+\beta-\frac{\beta b}{k}\right)-a^2\beta^3 e\left(1+\frac{a\theta}{\alpha}\right)+2a\beta^2(d+E_2)\left(\frac{a\beta e\theta}{\alpha}+b\right) - \frac{a\theta\beta^2(d+E_2)^2}{\alpha}(\beta+b)-3E_1\alpha\beta^2 b^2,$$

$$B_7 = 3\alpha r\beta^3 b^2-\frac{\alpha r\beta^3 b^3}{k}-a^2\beta^3 eb+2a\beta^3 b(d+E_2)\left(\frac{\theta(ae-1)+\alpha}{\alpha}\right) - 3E_1\alpha\beta^3 b^2\big(1-\theta(d+E_2)\big)-E_1\theta ea\beta^3 b^2,$$

$B_8 = a\beta^3 b^2(d+E_2)\left(\frac{\alpha-\theta}{\alpha}\right)-E_1\beta^3 b^2\big(\alpha-b\theta(d+E_2)\big).$

So, by Descartes rule of sign, we observe that, Eq. (5) has at least one positive root provided that

$$B_8 > 0. \tag{6}$$

Therefore, the interior equilibrium point y_2 exists if condition (6) is satisfied and the next condition met

$$(d+E_2) < \frac{ea\beta x^*}{(x^{*2}+\beta x^*+\beta b)}. \tag{7}$$

Now, we analyse the stability of all the equilibrium points by computing the Jacobian matrix J of system (1) at $y=(x,z)$.

For the point $y_0=(0,0)$,

$$J(y_0) = \begin{bmatrix} r-E_1 & 0 \\ 0 & -(d+E_2) \end{bmatrix}. \tag{8}$$

Clearly, the eigenvalues of $J(y_0)$ can be written as $\lambda_{01}=r-E_1$ and $\lambda_{02}=-(d+E_2)<0$. Therefore, y_0 is locally asymptotically stable (LAS) if the next condition met

$$r-E_1<0. \tag{9}$$

The Jacobian matrix at $y_1=\left(\frac{k(r-E_1)}{r},0\right)$ can be written as follows:

$$J(y_1) = \begin{bmatrix} -(r-E_1) & -r\theta\hat{x}+\frac{r\theta\hat{x}^2}{k}-\frac{a\beta\hat{x}}{\hat{x}^2+\beta\hat{x}+\beta b} \\ 0 & \frac{ea\beta\hat{x}}{\hat{x}^2+\beta\hat{x}+\beta b}-(d+E_2) \end{bmatrix}. \tag{10}$$

We observe that the eigenvalues of $J(y_1)$ can be written as $\lambda_{11}=-(r-E_1)<0$ and $\lambda_{12}=\frac{ea\beta\hat{x}}{\hat{x}^2+\beta\hat{x}+\beta b}-(d+E_2)$. Hence, y_1 is (LAS) provided the following condition holds

$$\frac{ea\beta\hat{x}}{\hat{x}^2+\beta\hat{x}+\beta b}<(d+E_2). \tag{11}$$

Finally, the Jacobian matrix at $y_2=(x^*,z^*)$ can be written as follows:

$$J(y_2) = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \tag{12}$$

where
$a_{11} = -\frac{rx^*}{k(1+\theta z^*)} + \frac{a\beta x^* z^*(2x^*+\beta)}{(x^{*2}+\beta x^*+\beta b)^2}$,
$a_{12} = -\frac{r\theta x^*}{(1+\theta z^*)^2} + \frac{rk\theta x^{*2}}{k^2(1+\theta z^*)^2} - \frac{a\beta x^*}{(x^{*2}+\beta x^*+\beta b)}$,
$a_{21} = \frac{eab\beta^2 z^* - ea\beta x^{*2} z^*}{(x^{*2}+\beta x^*+\beta b)^2}$, and $a_{22} = -\alpha z^*$.
The characteristic equation of $J(y_2)$ is computed by

$$\lambda^2 - T\lambda + D = 0, \tag{13}$$

where
$T = -\frac{rx^*}{k(1+\theta z^*)} + \frac{a\beta x^* z^*(2x^*+\beta)}{(x^{*2}+\beta x^*+\beta b)^2} - \alpha z^*$,
$D = \left(-\frac{rx^*}{k(1+\theta z^*)} + \frac{a\beta x^* z^*(2x^*+\beta)}{(x^{*2}+\beta x^*+\beta b)^2}\right)(-\alpha z^*) - \left(-\frac{r\theta x^*}{(1+\theta z^*)^2} + \frac{rk\theta x^{*2}}{k^2(1+\theta z^*)^2} - \frac{a\beta x^*}{(x^{*2}+\beta x^*+\beta b)}\right)\left(\frac{eab\beta^2 z^* - ea\beta x^{*2} z^*}{(x^{*2}+\beta x^*+\beta b)^2}\right)$.
Therefore, $T < 0$ and $D > 0$ if the following conditions hold

$$\frac{a\beta x^* z^*(2x^*+\beta)}{(x^{*2}+\beta x^*+\beta b)} < \frac{rx^*}{k(1+\theta z^*)} + \alpha z^*. \tag{14}$$

$$G_1 - G_2 > 0, \tag{15}$$

where
$G_1 = \frac{r\alpha z^* x^*}{k(1+\theta z^*)} + \frac{r\theta eab\beta^2 z^* x^*}{(1+\theta z^*)^2(x^{*2}+\beta x^*+\beta b)^2} + \frac{r\theta ea\beta x^{*4} z^*}{k(1+\theta z^*)^2(x^{*2}+\beta x^*+\beta b)^2} + \frac{ea^2 b\beta^3 z^* x^*}{(x^{*2}+\beta x^*+\beta b)^3}$.
$G_2 = \frac{a\alpha\beta x^* z^{*2}(2x^*+\beta)}{(x^{*2}+\beta x^*+\beta b)^2} + \frac{r\theta ea\beta x^{*3} z^*}{(1+\theta z^*)^2(x^{*2}+\beta x^*+\beta b)^2} + \frac{r\theta eab\beta^2 z^* x^{*2}}{k(1+\theta z^*)^2(x^{*2}+\beta x^*+\beta b)^2} + \frac{ea^2\beta^2 x^{*3} z^*}{(x^{*2}+\beta x^*+\beta b)^3}$.
Hence, the interior equilibrium point $y_2$is (LAS) provided that conditions (14)-(15) hold.

4. Persist of system (1)

The persistence of system (1) is studied. Its known that the existence of any system continues if and only if all species exist for all time, which means that the system (1) survives if the solution does not have an omega limit set in the boundary planes.
Now, we use the Dulac function to determine the potential of periodic dynamics in the $int. R_+^2$ of the $xz-$ plane.
Let $R(x,z) = \frac{1}{xz}$ be the Dulac function. Clearly, the function $R(x,z) > 0$ and it is a continuous function in the $int. R_+^2$ of the $xz-$ plane.
Moreover, we have
$\triangle(x,z) = -\frac{r}{kz(1+\theta z)} + \frac{a\beta(2x+\beta)}{(x^2+\beta x+\beta b)^2} - \frac{\alpha}{x}$.
Then $\triangle(x,z)$ doesn't identically to zero in the $int. R_+^2$ of the $xz-$ plane and it doesn't change sign under one of the next conditions:

$$\frac{a\beta(2x+\beta)}{(x^2+\beta x+\beta b)^2} > \frac{r}{kz(1+\theta z)} + \frac{\alpha}{x} \tag{16a}$$

or

$$\frac{a\beta(2x+\beta)}{(x^2+\beta x+\beta b)^2} < \frac{r}{kz(1+\theta z)} + \frac{\alpha}{x}\,. \tag{16b}$$

Note that, by applying the Bendixson-Dulac criterion, there's no periodic in the $int. R_+^2$ of the $xz-$plane for every trajectories that satisfy condition (16a) or condition (16b). Then by using the Poincare-Bendixon theorem, y_2 is a unique equilibrium point in the $int. R_+^2$ of the $xz-$plane and it will be a globally asymptotically stable (GAS) when its LAS.
Theorem (2): System (1) is uniformly persistent provided the following condition hold

$$d + E_2 < \frac{ea\beta\hat{x}}{\hat{x}^2+\beta\hat{x}+\beta b}\,. \tag{17}$$

Proof: Define the following function $\rho(x,z) = x^{\tau_1} z^{\tau_2}$, where $\tau_i, \forall i = 1,2$ are positive constants. Evidently, $\rho(x,z) > 0, \forall\, (x,z) \in int.R_+^2$ and $\rho(x,z) \to 0$ if $x \to 0$ or $z \to 0$. Let $\varphi(x,z) = \frac{\rho'(x,z)}{\rho(x,z)}$ then

$\varphi(x,z) = \tau_1\left(\frac{r}{1+\theta z}\left(1-\frac{x}{k}\right) - \frac{a\beta z}{x^2+\beta x+\beta b} - E_1\right) + \tau_2\left(\frac{ea\beta z}{x^2+\beta x+\beta b} - d - \alpha z - E_2\right)$.

Then we have that

$\varphi(y_0) = \tau_1(r - E_1) + \tau_2\big(-(d+E_2)\big)$,

$\varphi(y_1) = \tau_2\left(\frac{ea\beta\hat{x}}{\hat{x}^2+\beta\hat{x}+\beta b} - (d+E_2)\right)$.

Clearly, by choosing τ_1 to be sufficiently large with respect to τ_2, this leads to $\varphi(y_0) > 0$ and $\varphi(y_1) > 0$ under condition (17). Then system (1) is uniformly persistent.

Theorem (3): Assume that y_0 is LAS, then its GAS in the $int.R_+^2$.

Proof: Consider the positive definite function $L_1(x,z) = x + \frac{1}{e}z$

Note that $L_1(0,0) = 0$ and $L_1(x,z) > 0$ for all $(x,z) \in R_+^2$ with $(x,z) \neq (0,0)$. Now directly calculations give that

$\frac{dL_1}{dt} = \frac{rx}{1+\theta z} - \frac{rx^2}{k(1+\theta z)} - E_1 x - \frac{\alpha}{e}z^2 - \left(\frac{d+E_2}{e}\right)z$.

$\frac{dL_1}{dt} \leq (r - E_1)x - \left(\frac{d+E_2}{e}\right)z$.

Hence, under condition (9), it is clear that $\frac{dL_1}{dt}$ negative definite. Thus L_1 is a Lyapunov function and y_0 is a GAS.

Theorem (4): Assume that y_1 is LAS, then its a GAS in the $int.R_+^2$ if the next condition met

$$\theta\hat{x}(2r - E_1) + \frac{\alpha\hat{x}}{b} < d + E_2. \tag{18}$$

Proof: Consider the function $L_2(x,z) = \int_{\hat{x}}^{x} \frac{n-\hat{x}}{n}dn + z$

Note that $L_2(\hat{x},0) = 0$ and $L_2(x,z) > 0$ for all $(x,z) \in R_+^2$ with $(x,z) \neq (\hat{x},0)$ and $x > 0$. Now directly calculations give that

$\frac{dL_2}{dt} \leq \frac{r\theta\hat{x}z}{1+\theta z} - \frac{r}{k(1+\theta z)}(x-\hat{x})^2 + \frac{r\theta\hat{x}xz}{k(1+\theta z)} - \frac{a\beta xz}{x^2+\beta x+\beta b}(1-e) + \frac{a\beta\hat{x}z}{x^2+\beta x+\beta b} - (d+E_2)z.$

$\frac{dL_2}{dt} \leq \frac{-r}{k(1+\theta z)}(x-\hat{x})^2 - \left((d+E_2) - \left(\frac{r\theta\hat{x}(k+x)}{k(1+\theta z)} + \frac{a\beta\hat{x}}{x^2+\beta x+\beta b}\right)\right)z.$

Thus, with condition (18), its clear that $\frac{dL_2}{dt}$ negative definite. Then L_2 is the Lyapunov function and y_1 is a GAS.

5. Numerical Analysis

This section involves a numerical analysis of the dynamics of the system (1). This study proposes to explain the effects of changing the values of parameters on the system (1) and to validate the theoretical outcomes. The numerical analysis is done using the set of values of parameters which is given in the following. Now, the set of hypothetical parameters as shown below

$$\begin{gathered} r = 1.6,\ \theta = 0.2,\ k = 10,\ a = 2,\ \beta = 0.8,\ b = 5, \\ E_1 = 0.2,\ e = 0.75,\ d = 0.01,\ \alpha = 0.01,\ E_2 = 0.1. \end{gathered} \tag{19}$$

The trajectories of system (1) is obtained with four various sets of initial conditions approaches asymptotically to $y_2 = (7.62, 2.40)$, as shown in Figure 1.

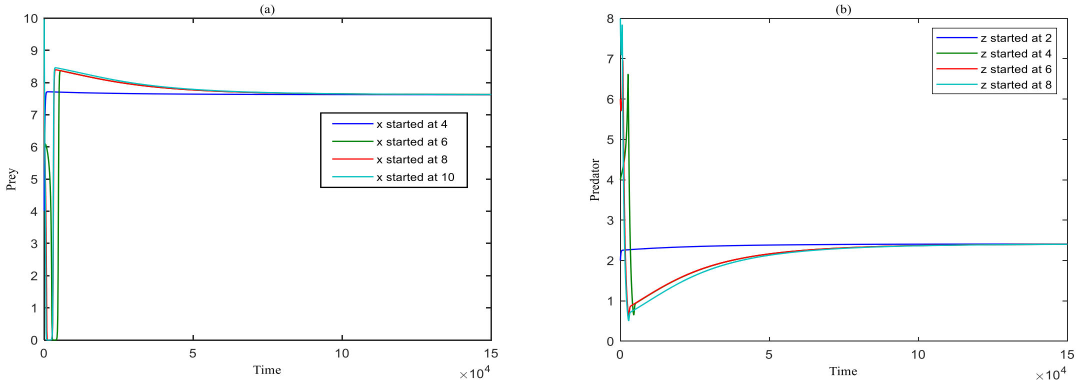

Figure 1: The trajectories of system (1) with data (19) with various initial points approach asymptotically to $y_2 = (7.62, 2.40)$. (a) Time series of prey's trajectories (b) Time series of predator's trajectories

Clearly, Figure 1 shows that y_2 is GAS. Now, we examine the effect of changing the parameter r , the growth rate, on the dynamics of the system (1) with the data given in Eq.(19), for $0.3 < r < 1.6$ then the system's trajectory approaches asymptotically to periodic dynamics. While $1.6 \leq r$ then the system's trajectory approaches asymptotically to y_2. see Figure 2.

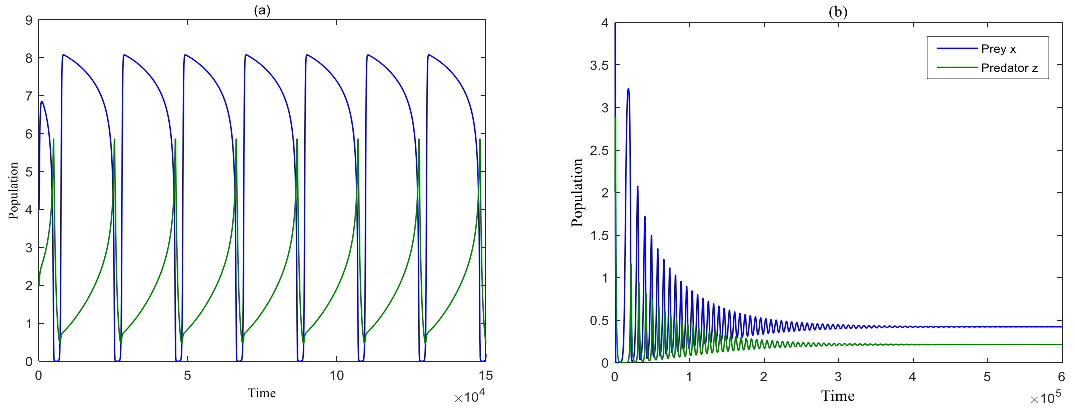

Figure 2: The trajectories of system (1) with data (19) and different values of r. (a) Time series with $r = 1$. (b) Time series with $r = 2$.

Now, the impact of fear rate θ with data (19) is investigated, for $0.01 < \theta \leq 0.2$ then the system's trajectory approaches asymptotically to y_2, while for $0.2 < \theta$ then the trajectory approaches to periodic attractor, see Figure 3.

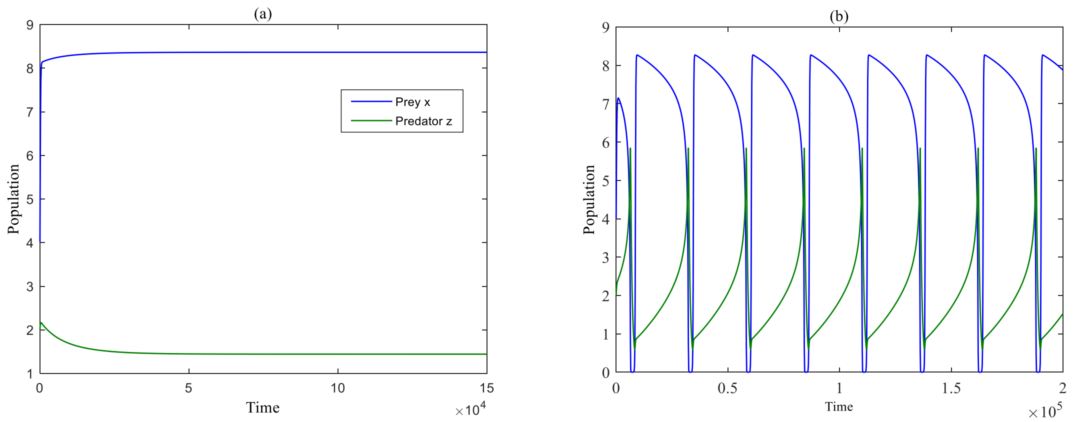

Figure 3: The trajectories of system (1) with data (19) and different values of θ. (a) Time series of the trajectory with $\theta = 0.1$ (b) Time series with $\theta = 0.3$.

The effect of the carrying capacity k is investigated in the ranges $1 < k \leq 4$, $5 \leq k < 10$ and $10 < k$, respectively. Its clear that the system's trajectory approaches to y_2, periodic dynamics and y_1, see Figure 4.

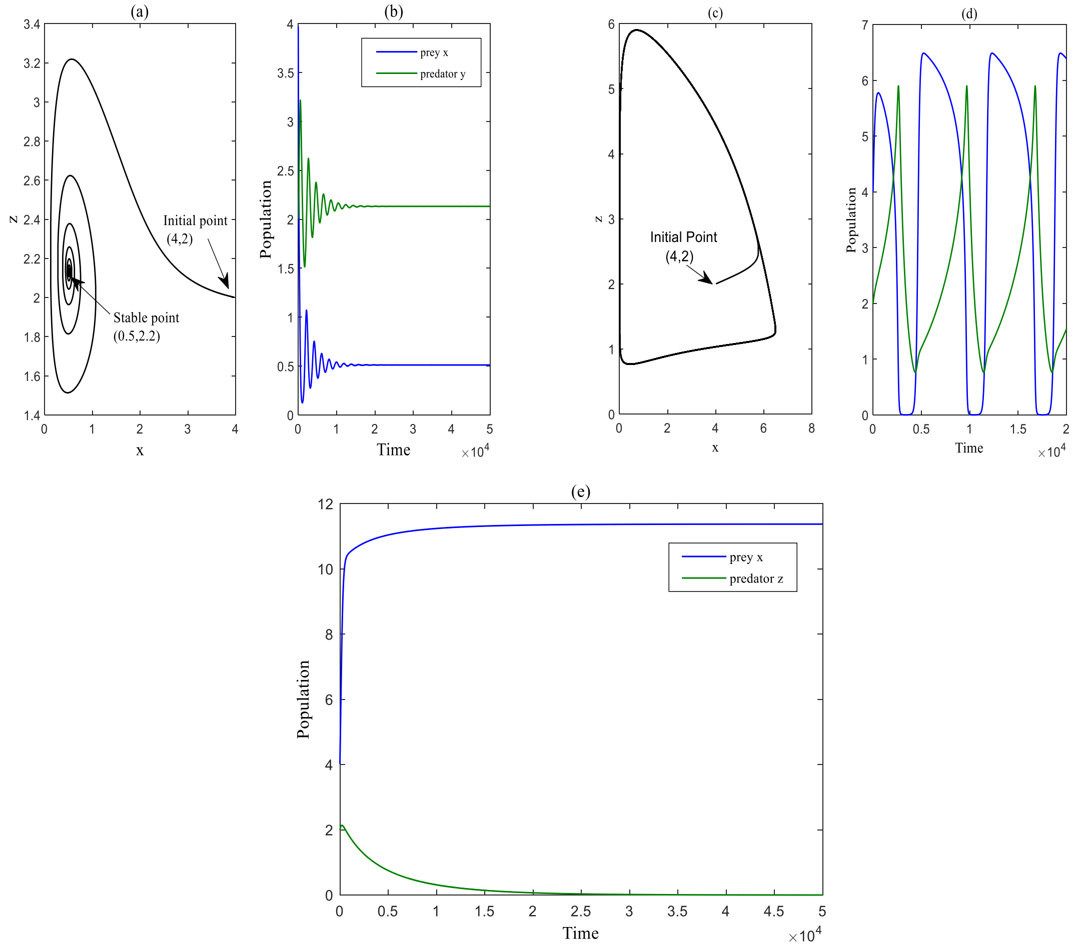

Figure 4: The trajectories of system (1) with data (19) and different values of k. (a) System (1) approaches to y_2 for $k = 3$. (b) Time series for (a). (c) Asymptotic stable periodic attracter for $k = 8$. (d) Time series for (c). (e) Time series with $k = 13$.

The impact of changing the parameter a, the attack rate, on the dynamical behavior is studied. For $0.1 \leq a < 2$, the trajectory approaches to y_1. Moreover, for $2 < a$ then the trajectory approaches to periodic dynamics as illustrated in Figure 5.

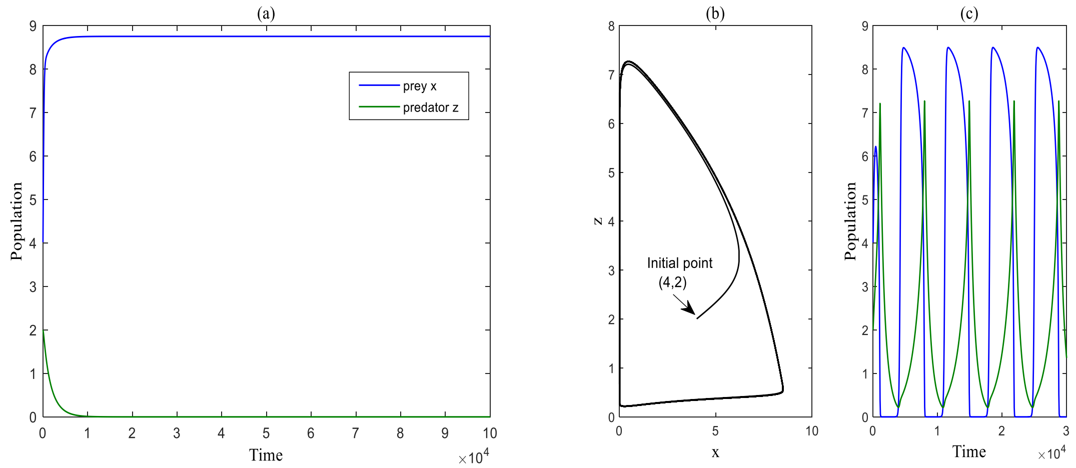

Figure 5: The trajectories of system (1) with data (19) and different values of a. (a) Time series with $a = 1$. (b) Periodic attracter for $a = 3$. (c) Time series for (b).

The influence of changing the direct measurement of the predator immunology from prey β on the dynamical behavior in different ranges is investigated. It is clear that for $0.1 \leq \beta < 0.8$, the trajectory approaches to y_1. Whilst for $0.8 < \beta$ then the trajectory approaches to periodic dynamics, see Figure 6.

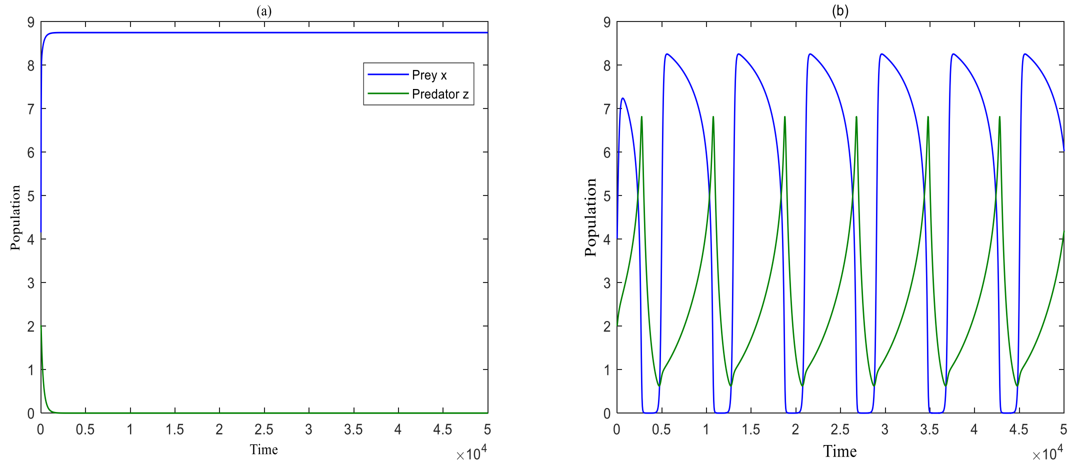

Figure 6: Time series of the trajectory of system (1) with data (19) and different values of β. (a) Time series with $\beta = 0.5$. (b) Time series with $\beta = 1$.

Now, the effect of varying the half saturation level b of the dynamical system is studied. In the range $1 \leq b < 5$, the trajectory approaches to periodic dynamics, while for $5 \leq b$, the trajectory approaches to y_2, as is illustrated in Figure 7.

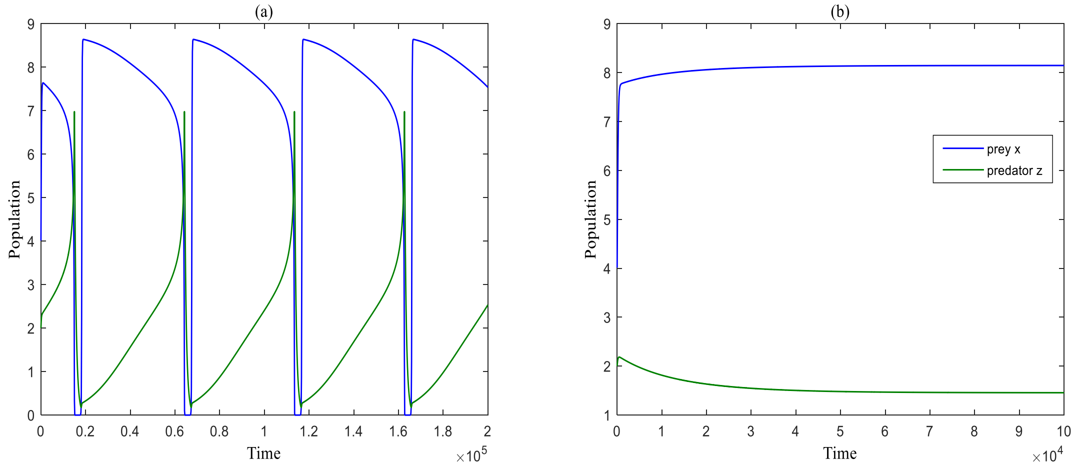

Figure 7: Time series of the trajectory of system (1) with data (19) and different values of b. (a) Time series with $b = 3$. (b) Time series with $b = 7$.

The influence of the harvesting efforts for the prey E_1 is investigated in the different ranges. It is clear that, for $0.01 \leq E_1 < 0.2$ the trajectory approaches to y_1, and for $0.2 < E_1 < 0.7$ the trajectory approaches to periodic dynamics. Also, for $0.7 \leq E_1 < 1.7$ the trajectory approaches to y_2. Now, to illustrate the obtained analytical outcomes that are given by condition (9) and for $1.7 \leq E_1$ leads to the harvesting rate (E_1) exceeds the growth rate (r) and the trajectory approaches to y_0, see Figure 8.

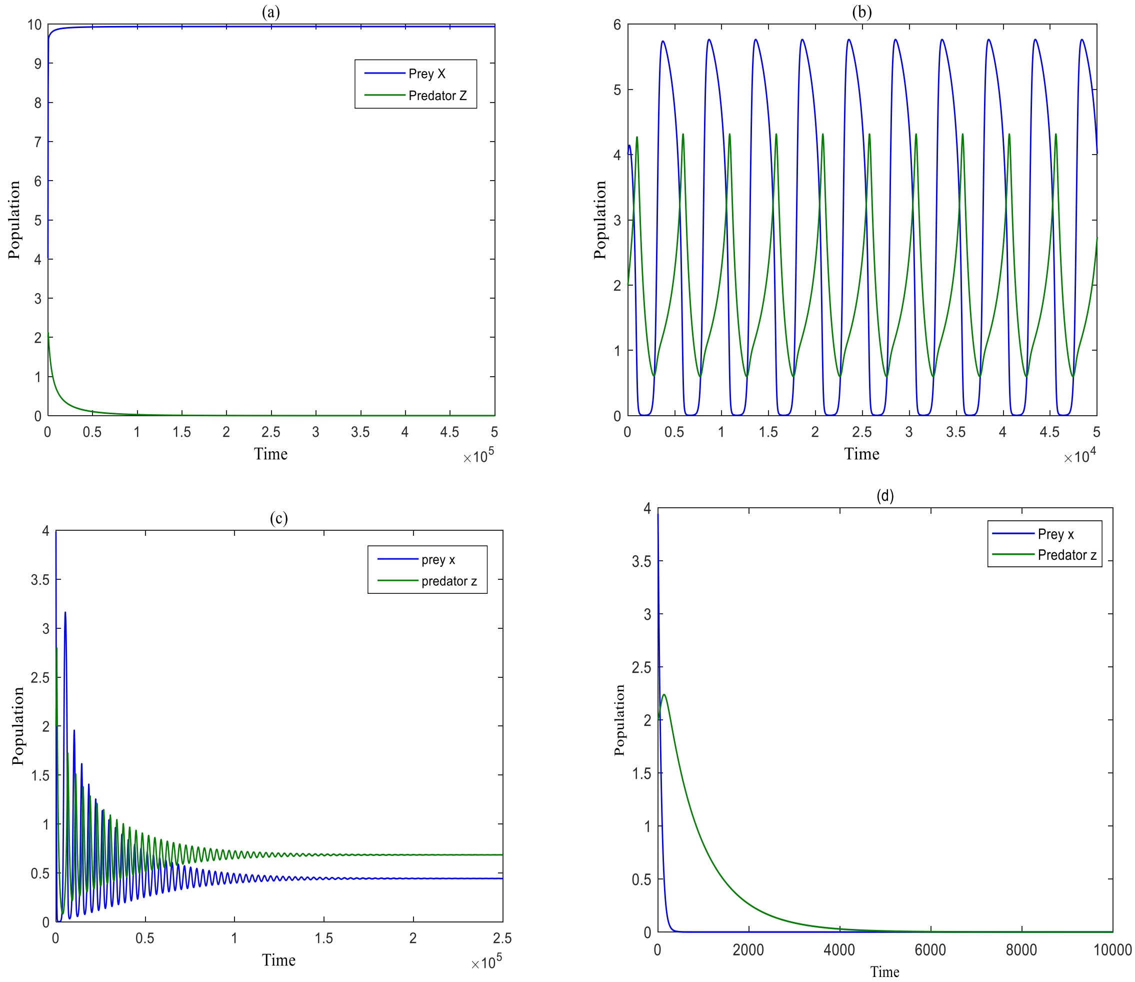

Figure 8: Time series of the trajectory of system (1) with data (19) and different values of E_1. (a) Time series $E_1 = 0.01$. (b) Time series $E_1 = 0.5$. (c) Time series with $E_1 = 0.8$. (d) Time series with $E_1 = 2$.

Now, the effect of changing the conversion rate e is studied. In the range $0.05 \leq e < 0.75$ the system's trajectory approaches asymptotically to y_1. Whilst, for $0.75 \leq e$ the trajectory approaches to periodic dynamics as its illustrated in Figure 9.

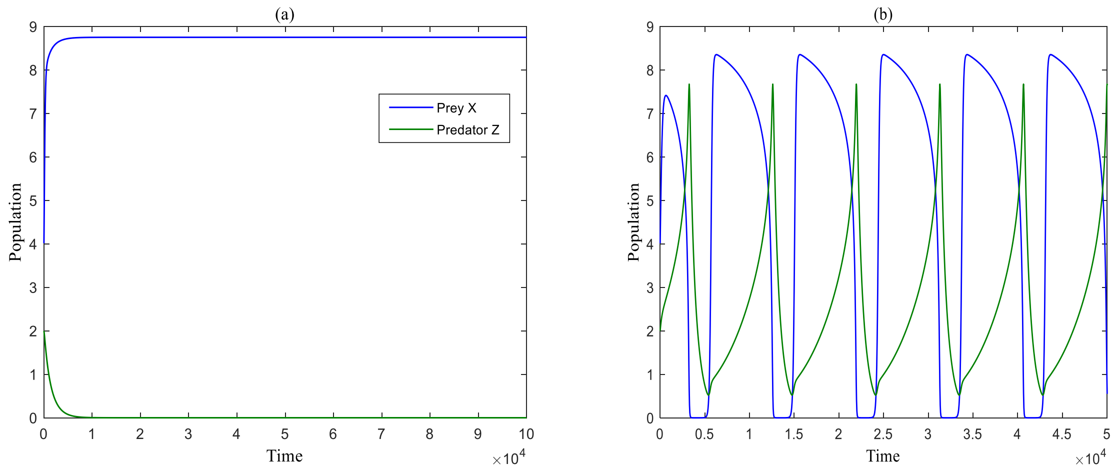

Figure 9: Time series of the trajectory of system (1) with data (19) and different values of e . (a) Time series with $e = 0.3$. (b) Time series with $e = 0.9$.

The influence of changing the death rate d on the dynamical behavior in various ranges is investigated. It is clear that for $0.001 \leq d < 0.01$ the trajectory approaches to periodic dynamics and for $0.01 < d$ the trajectory approaches to y_1 , see Figure 10.
\

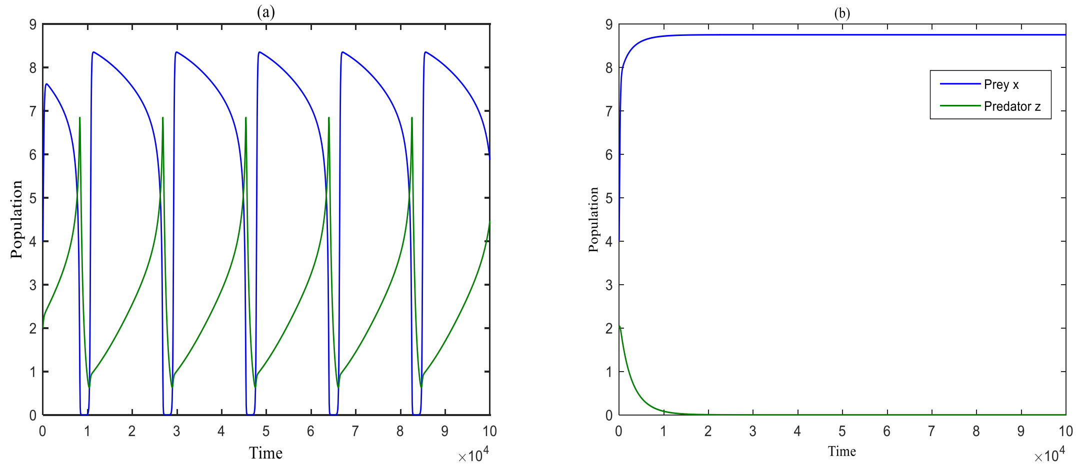

Figure 10: Time series of the trajectory of system (1) with data (19) and different values of d . (a) Time series with $d = 0.005$. (b) Time series with $d = 0.05$.

The effect of the intra-specific competition rate α is investigated in the different ranges. For $0.001 \leq \alpha < 0.01$ the trajectory approaches to periodic dynamics. Furthermore, for $0.01 < \alpha$ the trajectory approaches to y_2, see Figure 11.

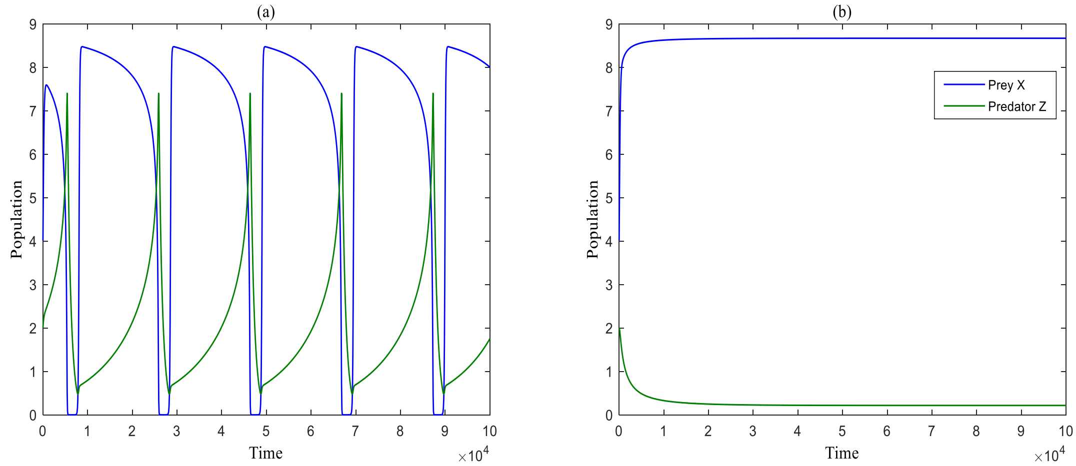

Figure 11: Time series of the trajectory of system (1) with data (19) and different values of α (a) Time series with $\alpha = 0.005$. (b) Time series with $\alpha = 0.05$.

The impact of changing the harvesting efforts for the predator E_2 on the dynamics system is investigated. In the ranges $0.01 \leq E_2 < 0.1$ and $0.1 < E_2$, respectively. Its shown that the trajectory approaches to periodic dynamics and y_1 as its illustrated Figure 12.

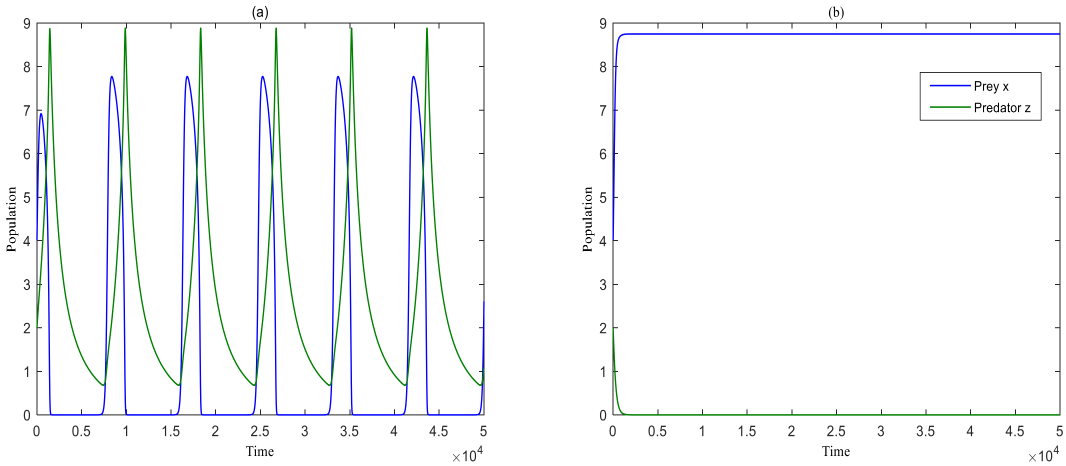

Figure 12: Time series of the trajectory of system (1) with data (19) and different values of E_2. (a) Time series with $E_2 = 0.01$. (b) Time series with $E_2 = 0.5$.

6. Conclusions

In this paper, the fear effect of the harvested prey-predator model with intra-specific competition is suggested and studied. The survival condition of the coexistence of each species in the system (1) is also established. Moreover, all equilibrium points of system (1) are investigated. The stability analysis (local and global) of all equilibrium points of system (1) are carried out. The Persist constraints are investigated. Numerical analysis is used to explain the impacts of changing the values of parameters on the system (1) and to validate the theoretical outcomes. By utilizing the set of hypothetical parameters that are given in Eq. (19), the following outcomes are obtained: (1) The solution to the system (1) approaches to $y_2 = (7.62, 2.40)$ when it begins from various initial points. (2) Its clear that any decrease in the parameter r below a given value leads to the system's trajectory approaches to the periodic dynamics. (3) An increase in the parameter θ above a specific value leads to the system's trajectory approaching to the periodic dynamics. (4) A decrease in the parameter k below a given value leads to the system's trajectory approaches to the periodic dynamics while increasing the parameter k above a given value lead to the trajectory approaches to y_1. The same observation will be got with decreasing and increasing in the parameters d, E_2. (5) A decrease in the parameter a below a given value leads to the system's trajectory approaches to y_1 while an increase the parameter a above a given value leads to the system's trajectory approaches to the periodic dynamics. The same scenario will be got with decreasing and increasing the parameters β, e. (6) A decrease in the parameter b below a given value leads to the system's trajectory approaches to the periodic dynamics. (7) A decrease in the parameter E_1 below a given value leads to the system's trajectory approaches to y_1 while an increase in the parameter E_1 above a given value leads to the system's trajectory approaches to the periodic dynamics and increasing more above a given value leads to y_2. However, if the harvesting rate E_1 exceeds the growth rate r, then the system's trajectory approaches to y_0. (8) A decrease in the parameter α below a given value leads to the system's trajectory approaches to the periodic dynamics while an increase in the parameter α above a given value leads to the system's trajectory approaches to y_2.

References

[1] N. Shigesada, K. Kawasaki and E. Teramoto, "Spatial segregation of interacting species," *J. Theor. Biol.*, vol. 79, pp. 83-99, 1979.

[2] X. Meng, R. Liu and T. Zhang, "Adaptive dynamics for a non-autonomous Lotka-Volterra model with size-selective disturbance," *Nonlinear Anal. Real World Appl.*, vol. 16, pp. 202-213, 2014.

[3] J.D. Murray, *Mathematical Biology: I. An Introduction*; Springer: Berlin/Heidelberg, Germany, 2002.

[4] N. H. Fakhry and R. K. Naji, "The Dynamics of a Square Root Prey-Predator Model with Fear," *Iraqi Journal of Science*, vol. 61, no. 1, pp: 139-146, 2020.

[5] K. Saleh, "A ratio-dependent predator-prey system with quadratic predator harvesting," *Asian Transactions on Basic & Applied Sciences*, vol. 02, no. 4, pp. 21–25, 2013.

[6] P. K. Tiwari, K. A. N. Al Amri, S. Samanta, Q. J. A. Khan and J. Chattopadhyaya, "A systematic study of autonomous and nonautonomous predator–prey models with combined effects of fear, migration and switching ," *Nonlinear Dynamics*, vol. 103, pp. 2125-2162. 2021.

[7] Md. M. Haque and S. Sarwardi, "Dynamics of a Harvested Prey–Predator Model with Prey Refuge Dependent on Both Species," *International Journal of Bifurcation and Chaos*, vol. 28, no. 12, 1830040 (16 pages), 2018.

[8] U. Kumar and P. S. Mandal, "Combine impact of cannibalism and Allee effect on the dynamics of a prey-predator model," *Journal of Biological Systems*, vol. 31, no. 4, pp. 1161-1234. 2023.

[9] A. M. A. Siddik, S. Toaha and A. M. Anwar, "Stability Analysis of Prey-Predator Model with Holling Type IV Functional Response and Infectious Predator," *Jurnal Matematika, Statistika dan Komputasi*, vol.17, no.2, pp.155-165, 2021.

[10] R. K. Naji, and R. N. shalan, " The Dynamics of Holling Type IV Prey Predator Model with Intra-Specific Competition," *Iraqi Journal of Science*, vol. 54, no. 2, pp: 386-396, 2013.

[11] S. Li, X. Wang, X. Li, and K. Wu, "Relaxation oscillations for Leslie-type predator–prey model with Holling Type I response functional function, " *Applied Mathematics Letters*, vol.120, no.3,107328, 2021.

[12] X. Liu, and L. Chen, "Complex dynamics of Holling type II Lotka–Volterra predator–prey system with impulsive perturbations on the predator, " *Chaos Solitons Fractals*, vol. 16, no. 2, pp: 311-320, 2003.

[13] Y. Wei, L. Huo, and H. He, "Research on Rumor-Spreading Model with Holling Type III Functional Response, " *Mathematics*, vol. 10, no. 4, 632, 2022.

[14] R. Taylor, *Predation, New York: Chapman & Hall*, 1984.

[15] A. Sha and J. Chattopadhyay, " Dynamical Study of Fear Effect in Prey Predator Model with Disease in Predator," *Journal of Biological Systems*, vol. 31, no. 4, pp. 1319-1340. 2023.

[16] J. Liu, P. Lv, B. Liu and T. Zhang, " Dynamics of a Predator-Prey Model with Fear Effect and Time Delay," *Complexity*, vol. 2021, Article ID 9184193, 16 pages, 2021.

[17] J. P. Suraci, M. Clinchy, L. M. Dill, D.Roberts and L. Y. Zanette, "Fear of large carnivores causes a trophic cascade," *Nat. Commun*, vol. 7, Article ID 10698, 2016.

[18] Y. Tian and H. M. Li, "The Study of a Predator-Prey Model with Fear Effect Based on State-Dependent Harvesting Strategy", *Complexity*, vol. 2022, Article ID 9496599, 19 pages, 2022.

[19] S. J. Majeed and S. F. Ghafel, "Stability Analysis of a Prey-Predator Model with Prey Refuge and Fear of Adult Predator," *Iraqi Journal of Science*, vol. 63, no. 10, pp: 4374-4387, 2022.

[20] T. K. Kar, "Modelling and analysis of a harvested prey–predator system incorporating a prey refuge," *Journal of Computational and Applied Mathematics*, vol. 185, pp. *19-33*, 2006.

[21] C. Raymond, Al. Hugo, and M. Kung'aro, "Modeling Dynamics of Prey-Predator Fishery Model with Harvesting: A Bioeconomic Model," *Journal of Applied Mathematics*, vol. 2019, Article ID 2601648, 13 pages, 2019.

[22] A. M. Abdellatif and D.K. Bahlool, "The Influence of fear on the dynamics of a prey-predator scavenger model with quadratic vesting," *Communications in Mathematical Biology and Neuroscience*, vol. 2022, no. 62, 2022.

[23] L. Chao, Z. Qingling, Z. Yue and D. Xiaodong, "Bifurcation and control in a differential-algebraic harvested prey-predator model with stage structure for predator, " *International Journal of Bifurcation and Chaos*, vol. 18, no. 10, pp. 3159-3168, 2008.

[24] H. A. Satar and R. K. Naji, "Stability and Bifurcation of a Prey-Predator-Scavenger Model in the Existence of Toxicant and Harvesting," *International Journal of Mathematics and Mathematical Sciences*, vol. 2019, Article ID 1573516, 17 pages, 2019.

[25] N. Bairagi, S. Chaudhuri and J. Chattopadhyay, "Harvesting as a disease control measure in an eco-epidemiological system-a theoretical study," *Mathematical Biosciences*, vol. 217, no. 2, pp. 134–144, 2009.

[26] R. Bhattacharyya and B. Mukhopadhyay, "On an ecoepidemiological model with prey harvesting and predator switching: local and global perspectives," *Nonlinear Analysis: Real World Applications*, vol. 11, no. 5, pp. 3824–3833, 2010. https://doi.org/10.1016/j.nonrwa.2010.02.012.

[27] S. Gakkhar and K. B. Agnihotri, "The dynamics of disease transmission in a prey predator system with harvesting of prey," *International Journal of Advanced Research in Computer Engineering and Technology*, vol. 1, no. 2, pp. 1–17, 2012.

[28] R.P. Gupta and P. Chandra, "Bifurcation analysis of modified Leslie–Gower predator–prey model with Michaelis–Menten type prey harvesting," *Journal of Mathematical Analysis and Applications*, vol. 398, no. 1, pp. 278–295, 2013.

[29] J. Cao and M. Xiao, "Hopf bifurcation and non hyperbolic equilibrium in a ratio-dependent predator–prey model with linear harvesting rate: Analysis and computation," *Mathematical and Computer Modeling*, vol. 50, no. 3, pp. 360–379, 2009.

[30] S. Al-Momen and R. K. Naji, "The Dynamics of Modified Leslie-Gower Predator-Prey Model Under the Influence of Nonlinear Harvesting and Fear Effect," *Iraqi Journal of Science*, vol. 63, no. 1, pp: 259-282, 2022.