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Power Law Fluid Gravity Fluctuations on Dual Component Convection in a Porous Channel in Presence of Throughflow

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Abstract

This study investigates the instability of power-law fluids in a porous channel, emphasizing dual component convection influenced by gravity variations and vertical throughflow. A single-term Galerkin approximation is employed to derive critical Rayleigh numbers and neutral stability curves. The findings reveal that the power-law index tends to destabilize the system, affecting the neutral stability curves, while buoyancy forces contribute to enhancing stability. The role of gravity variations is highlighted as a critical factor, with exponential-type fluctuations proving more effective at achieving stability compared to linear-type fluctuations. Overall, the study emphasizes that key parameters, including the power-law index, buoyancy force, and gravity variations, have a substantial impact on the system's stability characteristics.

Keywords: Variable gravity field; Non-Newtonian fluid; Buoyancy ratio; Throughflow; Lewis number.

1. Introduction

In engineering practice, non-Newtonian fluids flowing both steadily and irregularly in porous media with gravity acting as the primary driving force has found many uses. Fluids with non-Newtonian rheology are now included in the study of heat and mass transport issues by several researchers. Many models have been proposed to explain the behavior of fluids that are not Newtonian. The power law model gained prominence among them all. Though this model is based on an empirical link between stress and velocity gradients, it has shown efficacy in non-Newtonian fluid experiments. Notably, works by Nield and Bejan [1] and Drazin and Reid [2] provide detailed information on the instability properties of Newtonian fluids. For non-Newtonian fluids, the investigation into hydrodynamic and thermoconvective instability holds significant relevance with applications. However, investigating the behavior of non-Newtonian fluids constitutes a more intricate area of research, with various models such as the power-law model, the viscoelastic fluid model, and the Maxwell fluid model commonly employed (see Dharmadhikari and Kale [3], Alves et al. [4], Sochi [5] and Wang and Tan [6]). By analyzing the convective instability, Barletta and Nield [7] studied power-law fluid. Furthermore, Alves and Barletta [8] studied the limiting conditions of these scenarios and expanded this problem to incorporate Robin thermal boundary conditions in order to explore the convective instability of the power-law fluid in detail. When a significantly destabilizing thermal gradient conflicts with a stabilizing solutal gradient,

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oscillatory instability occurs, as shown by Nield [9]. Chen and Chen [10] elucidated the phenomenon of double-diffusive fingering in a fluid-saturated porous media, utilizing the Galerkin-finite difference method to address the boundary value problem. Wang and Tan [11] explored the double diffusion impact on Maxwell fluid model permeable bed. In the presence of Soret and Dufour effects, Mahdy [12] examined the mixed convection, Hussain et al. [13] examined the double diffusive mixed convection for porous cavity with magnetic field and Srinivasacharya and Reddy [14] studied the power-law fluid in two component natural convection porous channel. Kumari and Murthy [15, 16] studied the power-law fluid in two component convection for porous channel in the presence of throughflow, heat generation and viscous dissipation. Loenko et al. [17] studied the Power-Law Fluid for natural convection with heat source for square cavity. Srinivasacharya and Dipak [18] presented the study on throughflow, Soret, viscous dissipation and gravity field impacts on two component convection in a porous channel. Hussaini et al. [19] examined the power-law fluid flow for thermal conductivity, chemical reaction and viscosity in porous walls. Kolsi et al. [20] examined the dual component convection for porous channel through wavy interface. Alex and Patil [21] examined how the configuration of a porous bed is affected by gravity fluctuation combined with internal heating. Yadav [22] studied magnetic field and throughflow effects on porous bed configuration. Mahajan and Tripathi [23] investigated gravity fluctuation's impact on thermosolutal convective flow stability. Gangadharaiah et al. [24] recently analyzed penetrative solutal convective motion with varying gravity and throughflow in a fluid layer and also, examined the temperature-dependent viscosity with changing gravity about cross-diffusive terms. Very recently, by considering the throughflow and gravity modulation with couple-stress, Gaikwad and Preeti [25] studied the double component convection for porous channel. Some important works related to the nanofluid in presence of rotation, entropy generation, Soret and Dufour effects see Ali et al. [26-31] and also, presence of suction effect and gyrotactic microorganisms see Li et al. [32], Reddy et al. [33], Reddy and Ali [34], Srinivas Reddy et al. [35], Ahmed et al. [36-37], and Zafar et al. [38].

A non-Newtonian power-law fluid, vertical throughflow, and gravity variations are all taken into consideration in this study's solution to the issue of the onset of salt-finger convection in a porous substrate. We analytically address the eigenvalue issue using the single-term Galerkin technique, which is based on linear stability analysis and investigated the system's complexities in response to two distinct kinds of gravity variations, namely linear and exponential.

2. Problem statement

The convective motion of an infinitely large porous layer saturated by Ostwald-de-Waele power-law fluid with throughflow vertical velocity w_0 . The layer is between two boundaries $z = 0$ and $z = d$, and the vertical temperature difference ΔT and concentration difference ΔC across the boundaries. The arrangement is illustrated in detail in Figure 1.

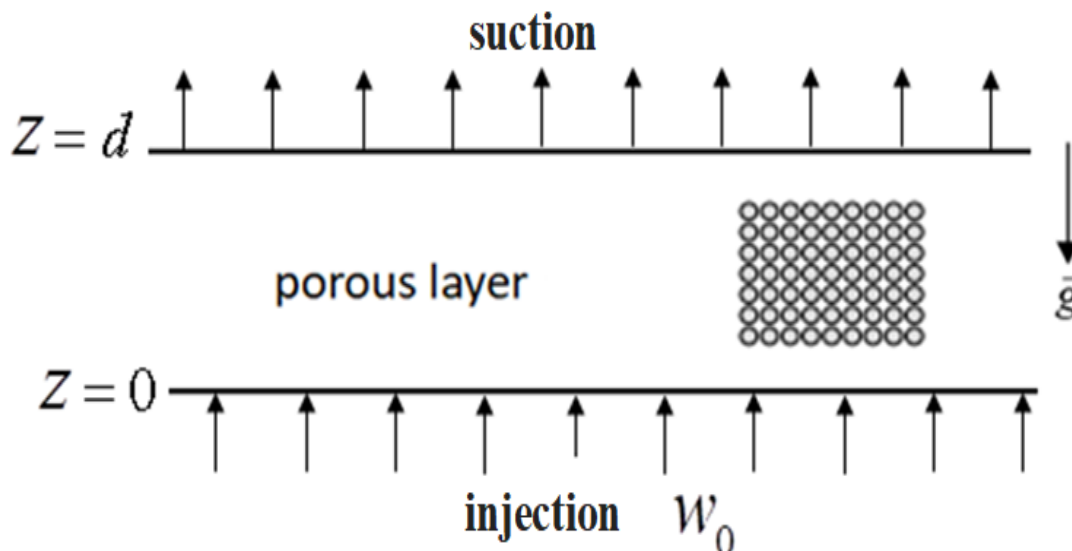


Figure 1: Physical configuration

Under the Boussinesq approximation, the current flow leading equations for these situations are (see Kairi and Murthy [39] and Shenoy [40])

$$\nabla \cdot \vec{V} = 0 \tag{1}$$

$$\frac{\mu^*}{K^*} |\vec{V}|^{(\eta-1)} \vec{V} = -\nabla p - \rho_0 [\beta_T (T - T_0) + \beta_C (C - C_0)] \vec{g} \tag{2}$$

$$\sigma \frac{\partial T}{\partial t} + (\vec{V} \cdot \nabla) T = \chi \nabla^2 T \tag{3}$$

$$\phi \frac{\partial C}{\partial t} + (\vec{V} \cdot \nabla) C = D_f \nabla^2 C. \tag{4}$$

The subsequent boundary situations are:

$$z = 0, w = w_0, T = T_0 + \Delta T, C = C_0 + \Delta C \tag{5}$$

$$z = d, w = w_0, T = T_0, C = C_0. \tag{6}$$

The non-dimensional quantities are,

$$\frac{1}{d} (x, y, z) = (x', y', z'), \quad \frac{1}{d} \vec{V} = \vec{V}', \quad \frac{x}{\sigma d^2} t = t', \quad \frac{T - T_0}{\Delta T} = T', \quad \frac{C - C_0}{\Delta C} = C'.$$

The above quantities are introduced to (1) to (4) and the prime notation is removed from every parameter and the curl operator is applied to (2), the non-dimensional form is

$$\nabla \cdot \vec{V} = 0 \tag{7}$$

$$\nabla \times |\vec{V}|^{(\eta-1)} \vec{V} = R [\nabla \times (T \times NC)] (1 + \delta h(z)) \tag{8}$$

$$\sigma \frac{\partial T}{\partial t} + (\vec{V} \cdot \nabla) T = \chi \nabla^2 T \tag{9}$$

$$\phi \frac{\partial C}{\partial t} + (\vec{V} \cdot \nabla) C = D_f \nabla^2 C. \tag{10}$$

Where, $R = \frac{\rho_0 g \beta_T \Delta T K^* d^n}{\mu^* \chi^n}$, $N = \frac{\beta_C \Delta C}{\beta_T \Delta T}$ are respectively, the non-Newtonian form of the Rayleigh number and the buoyancy ratio.

The fundamental solution to the governing Equation (7) to (10) is provided as

$$u_b = v_b = 0, w_b = w_0, T_b = \frac{(1 - e^{Pe z - Pe})}{(1 - e^{-Pe})} \text{ and } C_b = \frac{(1 - e^{Le Pe z - Le Pe})}{(1 - e^{-Le Pe})}. \tag{11}$$

The equations that describe the problem of thermal convection in power-law fluid can be expressed in a non-dimensionalized form are

$$(D^2 - \eta a^2) \hat{W} = \lambda a^2 (\hat{\Theta} + N \hat{\psi}) (1 + \delta h(z)) \tag{12}$$

$$(D^2 - a^2 - Pe D) \hat{\Theta} + \hat{W} f_T(z) = 0 \tag{13}$$

$$(D^2 - a^2 - Le Pe D)\widehat{\psi} + Le Pe \widehat{W} f_C(z) = 0. \quad (14)$$

The linearized boundary conditions are:

$$\widehat{W} = \widehat{\Theta} = \widehat{\psi} = 0 \quad \text{at } z = 0 \quad \text{and } z = 1. \quad (15)$$

Where, $Le = \frac{\chi}{D_f}$ is the Lewis number, $Pe = \frac{W_0 d}{\chi}$ is the Peclet number and $f_T(z) = \frac{Pe \cdot e^{Pe z}}{(e^{Pe} - 1)}$, $f_C(z) = \frac{Le \cdot Pe \cdot e^{Le \cdot Pe \cdot z}}{(e^{Le \cdot Pe} - 1)}$, $\lambda = \frac{R}{|Pe|(\eta - 1)}$.

3. Technique of solution

Obtaining a numerical solution is accomplished via the use of the Galerkin-type weighted residuals technique, in which three variables, \widehat{W} , $\widehat{\Theta}$ and $\widehat{\psi}$ are considered as

$$\widehat{W} = \sum_{i=1}^N a_i \widehat{W}_i, \quad \widehat{\Theta} = \sum_{i=1}^N b_i \widehat{\Theta}_i \quad \text{and} \quad \widehat{\psi} = \sum_{i=1}^N c_i \widehat{\psi}_i \quad \text{for } N = 1, 2, 3 \dots \dots \quad (16)$$

where a_i , b_i and c_i are constants. The functions, \widehat{W} , $\widehat{\Theta}$ and $\widehat{\psi}$ are projected as

$$\widehat{W}_i = \widehat{\Theta}_i = \widehat{\psi}_i = \sin(\pi i z). \quad (17)$$

The following matrix equations may be found by substituting (17) into (12)-(14) and integrating by taking the limit between $z = 0$ and $z = 1$.

$$\begin{bmatrix} -(n^2 \pi^2 + a^2) & RP_1 & NP_1 \\ P_2 & -(n^2 \pi^2 + a^2) & -(n^2 \pi^2 + a^2) \\ R_s P_3 Le & -Le(n^2 \pi^2 + a^2) & -(n^2 \pi^2 + a^2) \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (18)$$

$$R = \frac{\Delta \{ \Delta^2 (Le - 1) + NP_1 (R_s P_3 - P_2 Le) \}}{P_1 \{ R_s P_3 Le - P_2 \}}. \quad (19)$$

Where $\Delta = \pi^2 + a^2$, $P_1 = \int_0^1 a^2 G(z) P_4 dz$, $P_2 = \frac{N}{Le} \int_0^1 P_4 a^2 G(z) f_T(z) dz$, $P_3 = \int_0^1 f_C(z) P_4 dz$ and $P_4 = \sin(\pi i z)$.

4. Results and discussion

The combined effects of a non-Newtonian power-law fluid, gravity variations, and vertical throughflow are taken into consideration in the issue of the onset of dual component convection in a porous bed. A numerical solution is obtained using the Galerkin-type weighted residuals method. Two varieties of gravity fluctuation, case (i): Linear $h(z) = -z$, and case(ii): Exponential $h(z) = -(e^z - 1)$, were taken into account.

Figure 2 displays the effect of the buoyancy ratio parameter N on the linear form of gravity fluctuation. In contrast to opposing buoyancy forces ($N = 1$), the graphic shows that the system is more stable when buoyancy force help ($N = -1$) is present. Figures 3 and 4 show the impact of the critical Rayleigh number R^c for stationary mode concerning the gravity parameter for various values of the power law index η for a linear-type gravity field with the presence of buoyancy force assistance ($N = -1$) and opposing buoyancy forces ($N = 1$). By analysing these impartial stability curves, it is noticed that when the η increases alongside the variable gravity parameter, the critical Rayleigh number R^c decreases. As a result, the configuration becomes less stable for a higher power law index. Conversely, however, when the gravity parameter rises, the critical Rayleigh number R^c also increases, suggesting that the gravity parameter contributes to stabilizing the system configuration. Further noticed that the system exhibits greater stability in the presence of buoyancy force assistance ($N = -1$) as compared to opposing buoyancy forces ($N = 1$). Similar behavior is noted for an

exponential-type gravity field (see Figures 5 and 6) and also noted that an exponential-type gravity field is more stable compared to linear-type gravity variation.

In this scenario, the Lewis number emerges as a pivotal factor primarily influencing the stabilization of the flow. Figure 7 shows that when the gravity parameter increases, the neutral stability curves go higher, especially for low Lewis numbers like $Le = 0.1$. This implies that augmenting the Lewis number contributes to the stabilization of the fundamental flow. Moreover, it is detected that the system achieves greater stability when buoyancy force assists the flow, in contrast to scenarios involving opposing buoyancy forces. This observed trend remains consistent for an exponential-type gravity field, as depicted in Figure 8. Additionally, it is noteworthy that an exponential-type gravity field demonstrates superior stability when compared to the linear-type gravity variation.

Figures 9 and 10 displays the impact of the buoyancy and throughflow element on the linear form of gravity fluctuation. The figure demonstrates that the system exhibits greater stability in the presence of buoyancy force assistance ($N = -1$) as compared to opposing buoyancy forces ($N = 1$). As the Peclet number Pe increases, the stability curves undergo a downward shift, expanding the region of instability. This shift is attributed to the enhanced shear rate of the fluid resulting from the increase Pe , which, in turn, reduces the apparent viscosity. The buoyancy provided by the assistance helps to speed up the development of convective instability. Furthermore, it is noted that the system achieves enhanced stability when buoyancy force assistance is present, as opposed to situations where opposing buoyancy forces are involved. This behavior holds true for an exponential-type gravity field, as depicted in Figures 11 and 12. Furthermore, it is noteworthy that an exponential-type gravity field exhibits superior stability when contrasted with linear-type gravity variation.

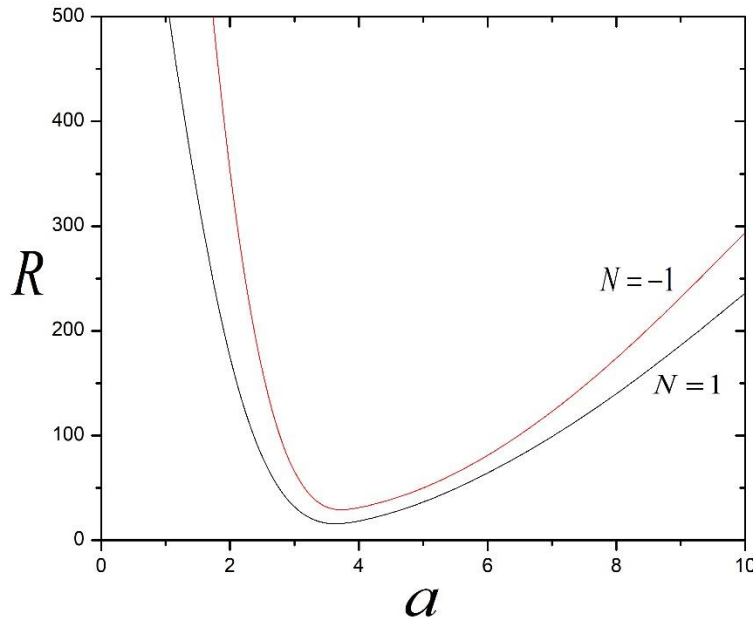


Figure 2: Effects of aiding and opposing buoyancy with $Pe = 5, Le = 0.1, \eta = 1.5$ and $\delta = 0.5$

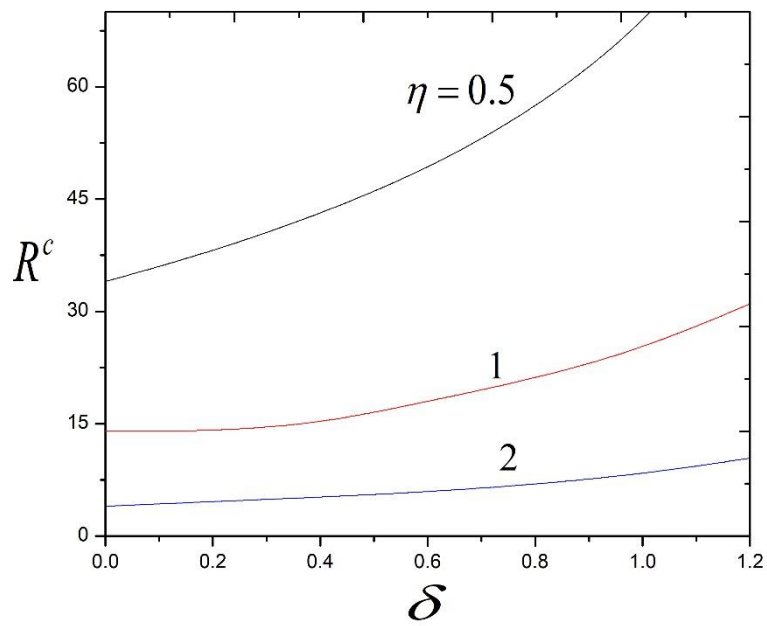


Figure 3: Effects of η with $Pe = 0.2, Le = 0.1, N = 1$ for linear gravity function.

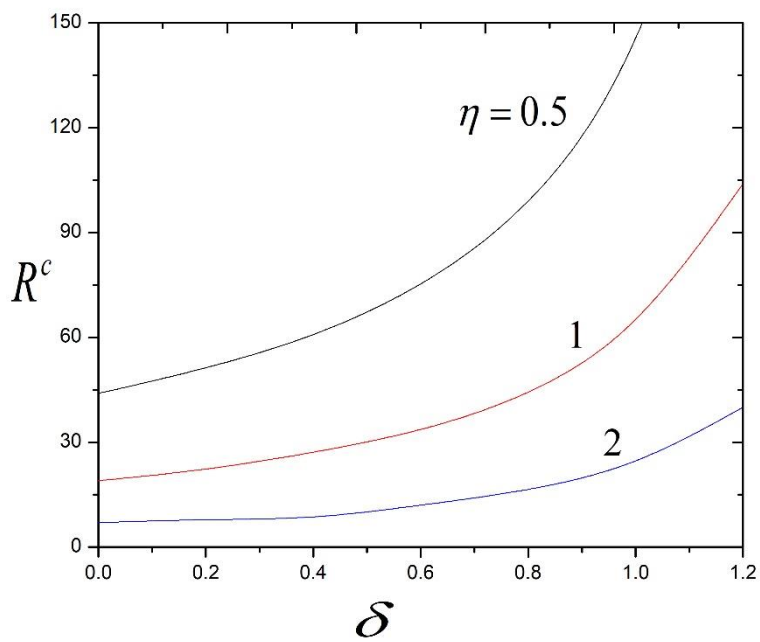


Figure 4: Impacts of η with $Pe = 0.2, Le = 0.1, N = -1$ for linear gravity function.

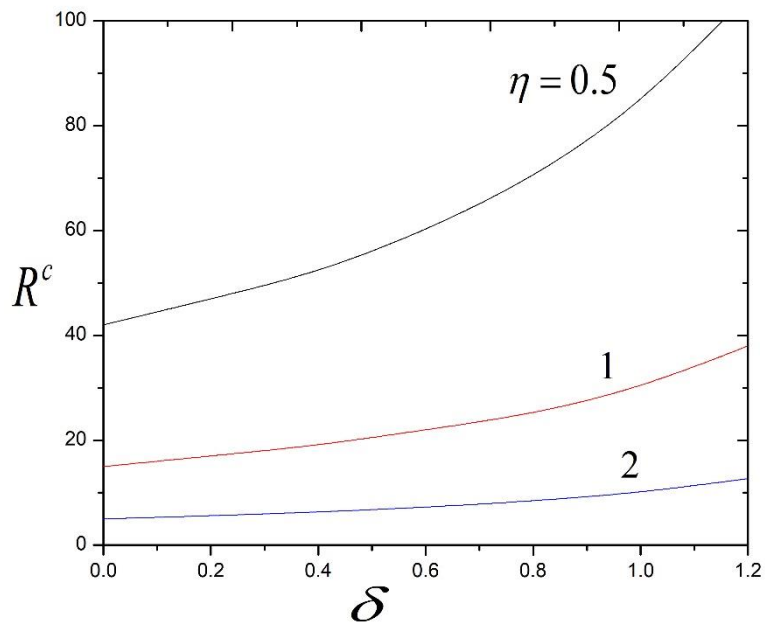


Figure 5: Effects of η with $Pe = 0.2, Le = 0.1, N = 1$ for exponential gravity function.

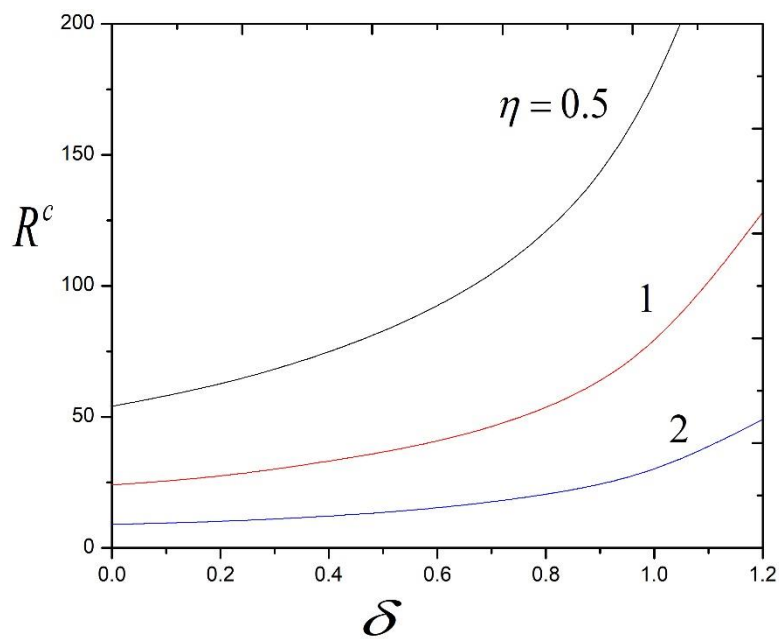


Figure 6: Impact of η with $Pe = 0.2, Le = 0.1, N = -1$ for exponential gravity function.

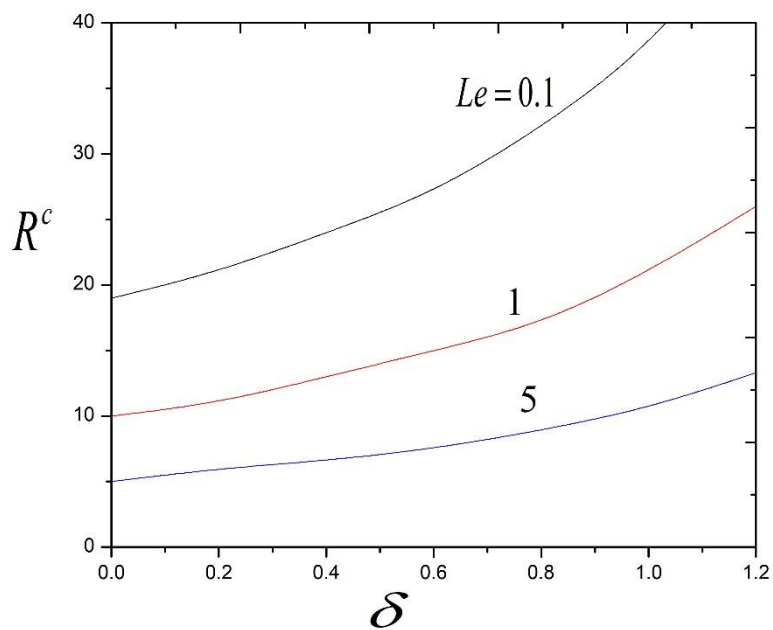


Figure 7: Impact of Le with $Pe = 0.2, \eta = 0.5, N = -1$ for linear gravity function.

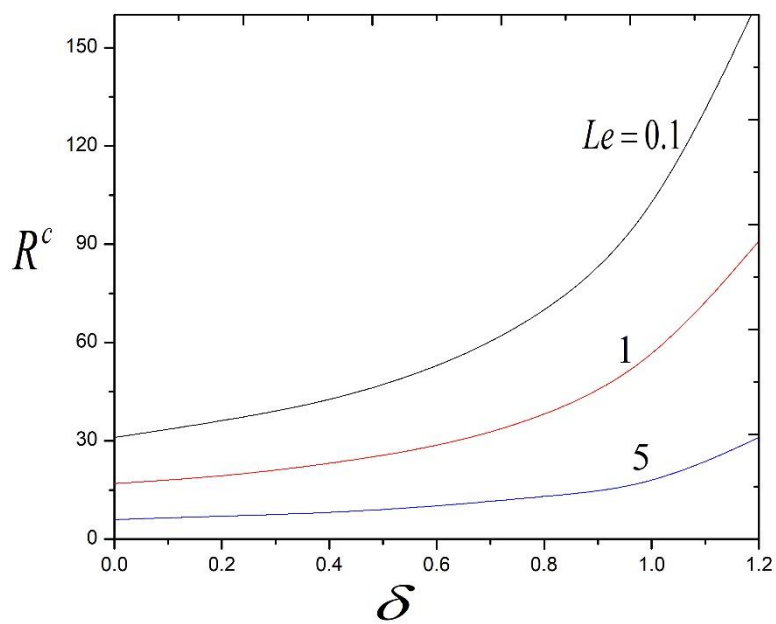


Figure 8: The effect of Le with $Pe = 0.2, \eta = 0.5, N = 1$ for exponential gravity function.

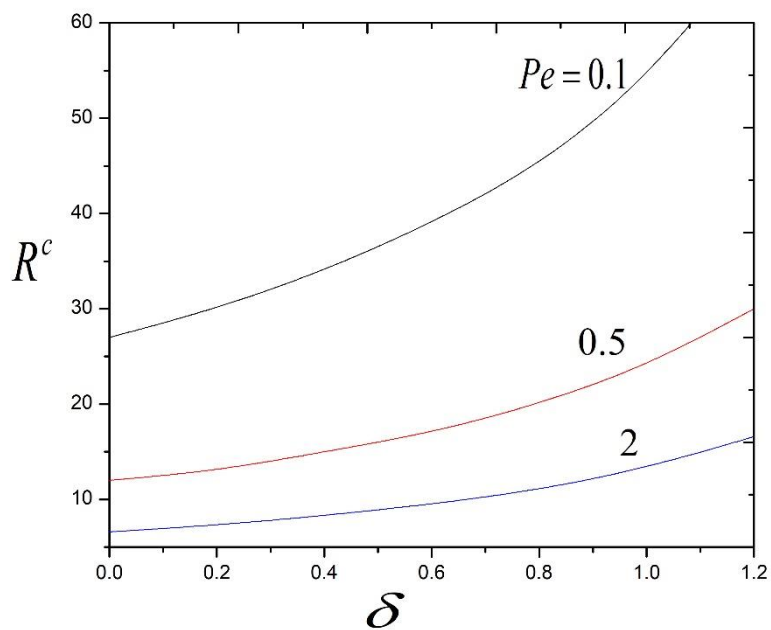


Figure 9: The effect of Pe with $Le = 0.1, \eta = 0.5, N = 1$ for linear gravity function.

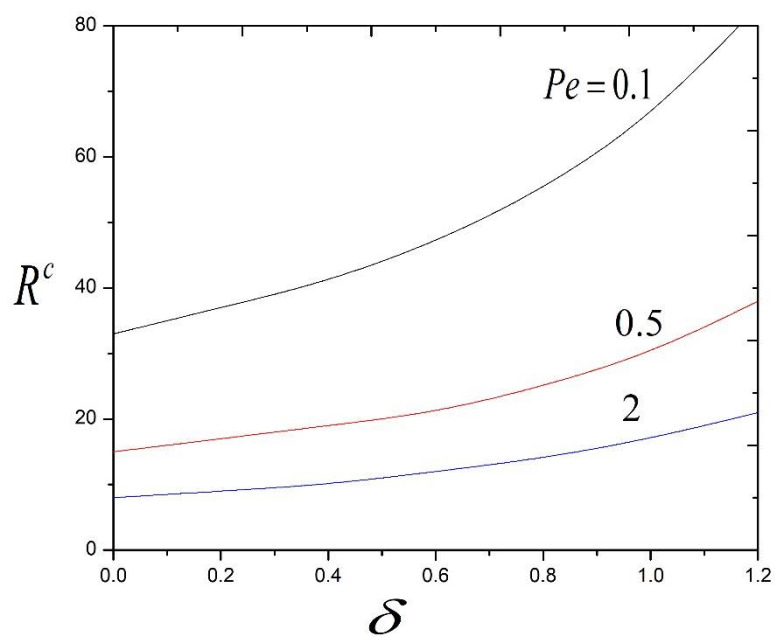


Figure 10: Effects of Pe with $Le = 0.1, \eta = 0.5, N = -1$ for linear gravity function.

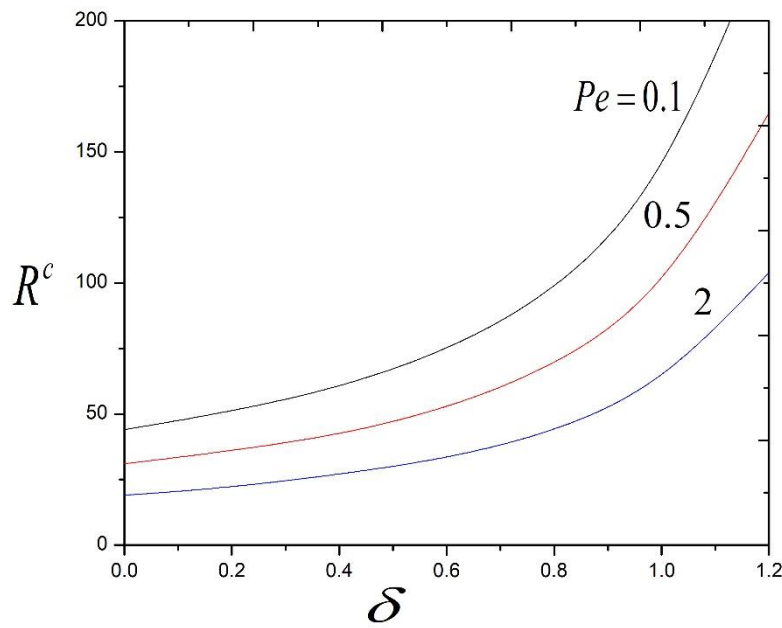


Figure 11: Impact of Pe with $Le = 0.1, \eta = 0.5, N = 1$ for exponential gravity function.

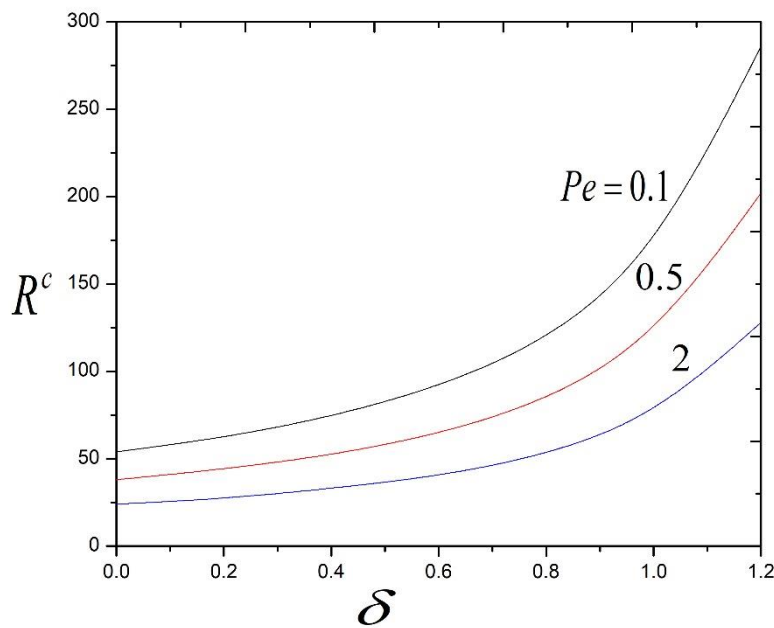


Figure 12: Impact of R^c verses δ for different values of Pe with $Le = 0.1, \eta = 0.5, N = -1$ for exponential gravity function.

5. Conclusions

Accounting for the combined effects of a non-Newtonian power-law fluid, vertical throughflow, and gravity variations, this work addresses the issue of the onset of dual component convection in a porous substrate. We use linear stability analysis to investigate the system's complexities as a function of both linear and exponential gravity fluctuations. The following are the most important takeaways from this study:

- The power law index parameter influences the stability system. The Rayleigh number decreases as the power law index parameter rises.
- The system configuration attains greater stability in the presence of buoyancy force assistance compared to scenarios involving opposing buoyancy forces.
- The variable gravity parameter has stabilizing impact whereas the Lewis number and throughflow parameter have a destabilizing impact on the configuration.
- Exponential-type gravity fluctuation has more stable than linear-type gravity fluctuation.

Nomenclature

a	Horizontal wavenumber	\vec{g}	Acceleration due to gravity
ΔT	Temperature difference	σ	Heat capacity ratio
ΔC	Concentration difference	T	Temperature
\vec{V}	Velocity vector	χ	Thermal diffusivity
μ^*	Viscosity	D_f	Solutal diffusivity
K^*	Permeability	C	Concentration
P	Pressure	ϕ	Porosity
ρ_0	Reference density	d	Depth
β_T	Thermal expansion coefficient of temperature	R_s	Solutal Rayleigh number
β_C	Thermal expansion coefficient of concentration	δ	Gravity parameter
T_0	Initial temperature	η	Power law index
C_0	Initial concentration	N	Buoyancy ratio
R	Non-Newtonian form of the Rayleigh number	Pe	Peclet number
R^C	Critical Rayleigh number	Le	Lewis number
w_0	Vertical velocity	D	$\frac{d}{dz}$

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