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A New Method for Estimating the Stress–Strength Model of Reliability System Based on the Generalized Inverse Pareto Distribution

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Abstract

The estimation of the stress–strength model is perhaps one of the most challenging concepts in reliability analysis. Therefore, in this paper, a new meta-heuristic algorithm (BAT algorithm) is used to estimate the Stress strength reliability system for the generalized inverse Pareto distribution. Furthermore, some statistical properties for generalized inverse Pareto distribution are derived. In a simulation study, the BAT algorithm and the classical Maximum Likelihood (MLE) method are compared using the Relative efficiency criterion. The results show that the BAT variant estimates the parameters more accurately than the MLE.

Keywords: BAT algorithm, Maximum Likelihood estimation, stress–strength model, reliability system, generalized inverse Pareto distribution

طريقة جديدة لتقدير نموذج الإجهاد– المتانه لموثوقية النظام بالاعتماد على توزيع باريتو المعمم المعكوس

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الخلاصه

يعد تقدير نموذج الاجهاد والمتانه أحد أكثر المفاهيم تحديًا في تحليل المعولية. ولذلك، في هذا البحث، تم استخدام خوارزمية جديدة استدلالية (خوارزمية الخفاش) لتقدير الإجهاد –المتانة لمعولية النظام لتوزيع باريتو المعكوس المعمم. علاوة على ذلك، تم اشتقاق بعض الخصائص الإحصائية لتوزيع باريتو المعكوس المعمم. في دراسة المحاكاة، تم مقارنة خوارزمية الخفاش وطريقة الامكان الاعظم (MLE) باستخدام معيار الكفاءه النسبية .تظهر النتائج أن خوارزمية الخفاش لتقدر المعلمات بشكل أكثر دقة من طريقة MLE

1. Introduction

Reliability is a broad term that focuses on the ability of a product to fulfill its intended function. The reliability of the stress-strength model has attracted many statisticians for several years because of its applicability in various and diverse fields such as engineering, quality control, and economics [1]. In the last thirty years, there have also been many applications to medical problems and clinical trials [2]. The term stress-strength (S-S) refers

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to a component that has a random strength X and is subjected to a random stress Y to evaluate reliability. The component fails when the stress applied to it exceeds the strength, while the component works when Y is less than X (Y<X). Various lifetime distributions were assumed by several researchers for the stress- strength random variates [3-8]. On the other hand, statistical distributions have long been employed in the assessment of semiconductor devices and product reliability [9]. Furthermore, many situations where both failure rate functions are useful in real-world applications.

On the other hand, the class of Pareto-based models is one of the well-known classes of distributions in statistical analysis. Specifically, in the stress–strength estimation, many contributions on postulating accurate models were recently published by considering Pareto models [10]. For the last few decades, improvement over standard distributions have become a common practice in statistical theory. In addition, a majority of applications in biological sciences, survey sampling, engineering sciences, life testing, medical research, and econometrics problems have been concerned in applying inverse distribution [11-14].

The inverse Pareto distribution (IPD) can be adapted to the various forms of failure rate functions often observed in medical studies, e.g. heart transplants, cancer, etc. The IPD may be suitable for analyzing such data [15]. In reliability engineering, [16] discussed the application of the IPD lifetime model using the air conditioning failure time of two airplanes. In addition, several researchers have described the popularity of the IPD life model. For example, [17] discussed various estimates of the reliability function properties using the application data of head and neck cancer and then compared the performance of the IPD lifetime model with some other existing lifetime models. In addition, [18] presented the S-S of reliability based on progressively censored data for the inverse Pareto distribution lifetime model. In [19] estimated the reliability function using progressively censored first failure data.

However, [20] presented the generalized inverted scale family distributions. They formulated new models by introducing a new shape parameter to the scale family of distributions. These distributions gave the flexibility in modeling complex data, and the results seemed genuine and quite sound [21-24]. Therefore, the generalized inverse Pareto distribution was introduced. However, the large number of parameters and the nonlinear model make the classical estimation method more difficult [25-26]. Additionally, Because of meta-heuristic algorithms have many advantages including the simplicity of implementation that is robust, reliable, and effective. Hence, the meta-heuristic algorithms were used to overcome these difficulties by hybridizing the BAT algorithm with the MLE method to estimate the S-S model of the reliability system based on generalized inverse Pareto distribution (GIP).
The rest of the paper is organized as follows: Section 2 presents general inverse Pareto distribution. Section 3 includes some statistical properties of GIPD. Sections 4 and 5 drive the Stress-Strength model in system reliability of GIPD and maximum likelihood estimation of P(Y<X), respectively. Section 6 describes BAT-MLE Algorithm. Section 7 offers a simulation study. Section 8 demonstrates the effectiveness of the proposed method through results. Finally, a conclusion is provided in Section 9.

2. General inverse Pareto distribution

Suppose X is a random variable following inverse Pareto distribution having probability density function (p.d.f) and the cumulative distribution function (c.d.f), respectively as follows;

$$g(y,\alpha,\beta) = \alpha\beta^{\alpha}\, y^{\alpha-1}. \quad (1)$$
$$G(y)=(\beta y)^{\alpha}. \quad (2)$$

We have used the generalization method of Mudholkar and Srivastava [27] for different distributions depending on the c.d.f to generalize the inverse Pareto distribution as follows:
$$F(y)=[(\beta y)^{\alpha}]^{\theta}=(\beta y)^{\alpha\theta}. \quad (3)$$
Thus, the p.d.f of generalize inverse Pareto distribution (GIPD)
$$f(x,\alpha,\beta,\theta) = \begin{cases} \alpha\theta\beta^{\alpha\theta}\, y^{\alpha\theta-1} & {}^{x>0}_{\beta,\alpha,\theta>0} \\ 0 & 0.w \end{cases}. \quad (4)$$
T
he Survival function of and Hazard function of GIPD are:
$$R(y)=1-F(y) =1- (\beta y)^{\alpha\theta}. \quad (5)$$
$$h(y)= \frac{f(y)}{R(y)} = \frac{\alpha\theta\beta^{\alpha\theta}\, y^{\alpha\theta-1}}{1-(\beta y)^{\alpha\theta}} \qquad y > 0, \quad \beta,\alpha,\theta > 0. \quad (6)$$

3. Statistical properties of GIPD

In this section, some statistical properties of GIPD were introduced as follows:

The moment generating function of GIPD

$$M_y(t) = E(e^{ty}) = \int_0^{\frac{1}{\beta}} e^{ty} \; f(y)dy = \int_0^{\frac{1}{\beta}} e^{ty} \; \alpha\theta\beta^{\alpha\theta} \; y^{\alpha\theta-1}\, dy.$$

By using Taylor′sseries of e^{ty}

$$\int_0^{\frac{1}{\beta}} \alpha\theta\beta^{\alpha\theta} \sum_{n=0}^{\infty} \frac{(ty)^n}{n!}\, y^{\alpha\theta-1}\, dy = \alpha\theta\beta^{\alpha\theta} \sum_{n=0}^{\infty} \int_0^{\frac{1}{\beta}} \frac{(t)^n}{n!}\, y^{\alpha\theta+n-1}\, dy$$
$$= \alpha\theta\beta^{\alpha\theta} \sum_{n=0}^{\infty} \frac{(t)^n}{n!} \left[\frac{y^{\alpha\theta+n}}{\alpha\theta+n}\right]_0^{\frac{1}{\beta}}$$
$$= \alpha\theta \sum_{n=0}^{\infty} \frac{(t)^n}{n!} \cdot \frac{1}{\beta^n\,(\alpha\theta+n)}.$$

The mean of GIPD

$$\mu_Y = E(y) = \int_0^{\frac{1}{\beta}} y\, f(y)dy$$
$$= \int_0^{\frac{1}{\beta}} y\alpha\theta\beta^{\alpha\theta}\, y^{\alpha\theta-1}\, dy$$
$$= \int_0^{\frac{1}{\beta}} \alpha\theta\beta^{\alpha\theta}\, y^{\alpha\theta}\, dy$$
$$= \alpha\theta\beta^{\alpha\theta} \left[\frac{y^{\alpha\theta+1}}{\alpha\theta+1}\right]_0^{\frac{1}{\beta}}$$
$$= \frac{\alpha\theta}{\beta(\alpha\theta+1)}.$$

The variance of GIPD

$$Var(y)=E(y^2) - (E(y))^2.$$
$$\text{As } E(y^2) = \int_0^{\frac{1}{\beta}} y^2\, f(y)dy$$
$$= \int_0^{\frac{1}{\beta}} y^2\, \alpha\theta\beta^{\alpha\theta}\, y^{\alpha\theta-1}\, dy$$
$$= \alpha\theta\beta^{\alpha\theta}\, \frac{(\frac{1}{\beta})^{\alpha\theta+2}}{\alpha\theta+2}$$
$$= \frac{\alpha\theta}{\beta^2\,(\alpha\theta+2)}.$$

Then, Var(y) = $E(y^2) - (E(y))^2$

$= \frac{\alpha\theta}{\beta^2\,(\alpha\theta+2)} - \frac{\alpha^2\,\theta^2}{\beta^2\,(\alpha\theta+1)^2}$

$= \frac{\alpha\theta}{\beta^2(\alpha\theta+2)(\alpha\theta+1)^2}$.

The median of GIPD is;

F(y) = $\frac{1}{2}$ ➔ y=$\frac{1}{2^{\frac{1}{\alpha\theta}}\ \beta}$

4. Stress-strength model in system reliability of GIPD

The system reliability in stress-strength single component model (R_{si}). Suppose X and Y are two random variables that follow the GIPD with parameters $(\alpha, \beta, \theta_1)$ and $(\alpha, \beta, \theta_2)$ as strength and stress, respectively. When θ_1, θ_2 are unknown parameters and α, β are known parameters. From the probability density function of random variables X and Y which are distributed as GIPD have the following forms;

$$f(x,\alpha,\beta,\theta_1) = \begin{cases} \alpha\theta_1\beta^{\alpha\theta_1}x^{\alpha\theta_1-1} & x>0 \\ 0 & 0.w \quad \beta,\alpha,\theta>0 \end{cases}$$

$$g(y,\alpha,\beta,\theta_2) = \begin{cases} \alpha\theta_2\beta^{\alpha\theta_2}y^{\alpha\theta_2-1} & y>0 \\ 0 & 0.w \quad \beta,\alpha,\theta_2>0 \end{cases}$$

And from the cumulative distribution function (c.d.f) of x and y have the form below:

$F(x,\alpha,\beta,\theta_1)=(\beta x)^{\alpha\theta_1}$.

G$(y,\alpha,\beta) = (\beta y)^{\alpha\theta_2}$.

The system reliability in the S-S model is defined as below:

$R_{si} = P(y < x) \qquad 0< y < x < \infty$

$=\int_0^\infty \int_y^\infty f(x)g(y)dxdy$

$=\int_0^\infty g(y).\bar{F}(y)\, dy$.

Since: F$(,\alpha,\beta,\theta_1) = \alpha\theta_2\beta^{\theta_2}y^{\alpha\theta_2}$, $\quad \bar{F}_{(x)}(y) = 1 - F_{(x)}(y) = 1 - (\beta y)^{\alpha\theta_1}$

$$R_{si} = \int_0^\infty [\alpha\theta_2\beta^{\theta_2}y^{\alpha\theta_2}(1 - (\beta y)^{\alpha\theta_1})]dy$$

$=\int_0^\infty \alpha\theta_2\beta^{\theta_2}y^{\alpha\theta_2} - \alpha\theta_2\beta^{\alpha(\theta_1+\theta_2)}.y^{\alpha(\theta_1.\theta_2)}\, dy$

$=\int_0^\infty \alpha\theta_2\beta^{\theta_2}y^{\alpha\theta_2}\, dy - \int_0^\infty \alpha\theta_2\beta^{\alpha(\theta_1+\theta_2)}.y^{\alpha(\theta_1+\theta_2)-1}dy$

$=1\text{-}\theta_2\int_0^\infty \alpha\frac{(\theta_1+\theta_2)}{(\theta_1+\theta_2)}.\beta^{\alpha(\theta_1+\theta_2)}.y^{\alpha(\theta_1+\theta_2)-1}dy$

$=1\text{-}\frac{\theta_2}{\theta_1+\theta_2}\int_0^\infty \alpha(\theta_1+\theta_2).\beta^{\alpha(\theta_1+\theta_2)}.y^{\alpha(\theta_1+\theta_2)-1}dy$

$=1\text{-}\frac{\theta_1}{\theta_1+\theta_2}$.

Therefore, $R_{si} = \frac{\theta_1}{\theta_1+\theta_2}$. (7)

5. Maximum likelihood estimation method (MLE) of P(Y<X)

Let $x_1, x_2, \ldots, x_n$ be a.r.s from the GIPD(α,β,θ_1), and $y_1, y_2, \ldots, y_m$ be a random sample from GIPD(α,β,θ_2) , where θ_1, *and* θ_2 are unknown parameters and α, β are known.

L=L$(x_i, y_j, \alpha, \beta, \theta_i)$

$= \prod_{i=1}^{n} f(x_i, \alpha,\beta,\theta_1) \, . \prod_{j=1}^{m} g(y_j,\alpha,\beta,\theta_2)$

$$= \alpha^n{\theta_1}^n(\beta^{\alpha\theta_1})^n\prod_{i=1}^{n}{x_i}^{\alpha\theta_1-1}.\ \alpha^n{\theta_2}^n(\beta^{\alpha\theta_2})^n\prod_{i=1}^{m}{y_i}^{\alpha\theta_2-1}.$$

$$\ln l = \ln[\alpha\theta_1\beta^{\alpha\theta_1}]^n + \ln\prod_{i=1}^{n} x_i^{\alpha\theta_1-1}. \quad \ln[\alpha\theta_2\beta^{\alpha\theta_2}]^m + \ln\prod_{j=1}^{m} y_i^{\alpha\theta_2-1}$$

$= \ \text{n}Ln\,\alpha + nLn\,\theta_1 + n\ln\beta^{\alpha\theta_1} + (\alpha\theta_1 - 1)\ln\prod_{i=1}^{n} x_i \quad + \quad mLn\,\alpha + mLn\,\theta_2 + m\ln\beta^{\alpha\theta_2} + (\alpha\theta_2 - 1)\ln\prod_{i=1}^{m} y_i$

$=\ln\alpha + n\ln\theta_1 + n\alpha\theta_1\ln\beta + (\alpha\theta_1)\sum_{i=1}^{n}\ln x_i - \sum_{i=1}^{n}\ln x_i \ + \ \text{m}\ln\alpha + m\ln\theta_2 + m\alpha\theta_2\ln\beta \ + (\alpha\theta_2)\sum_{j=1}^{m}\ln y_i - \sum_{j=1}^{m}\ln y_i$

$\frac{\partial \ln L}{\partial\theta_1} = \ 0 + \frac{n}{\theta_1} + n\alpha\ln\beta \ + \alpha\sum_{i=1}^{n}\ln x_i = 0.$

The MLE for the unknown parameters (θ_1, θ_2) of GIPD

$\theta_1 \ = \frac{-n}{n\alpha\ln\beta + \alpha\sum_{i=1}^{n}\ln x_i}$

$\widehat{\theta_1} \ = \frac{-n}{\alpha(n\ln\beta + \sum_{i=1}^{n}\ln x_i)}$

Then, we derivative with respect to θ_2 and equate to zero we get:

$\widehat{\theta_2} \ = \frac{-m}{\alpha(m\ln\beta + \sum_{j=1}^{m}\ln y_i)}.$

By substituting Equation (7) from Equation (5), we get

$$\hat{R}_{MLE} = \frac{\hat{\theta}_2}{\hat{\theta}_1 + \hat{\theta}_2}. \tag{8}$$

6. BAT-MLE Algorithm

The BAT algorithm was a nature-inspired algorithm belonging to the SI family. Xin-She-Yang was the first to introduce the BAT algorithm in 2010 [28]. BAT works with the echolocation of microbats and uses the echo of bats for foraging. Their path is detected at night by emitting a sound signal called sonar echolocation and using this signal to detect objects or obstacles in their environment [29]. The following chart presented the detailed steps for the standard bat algorithm as follows:

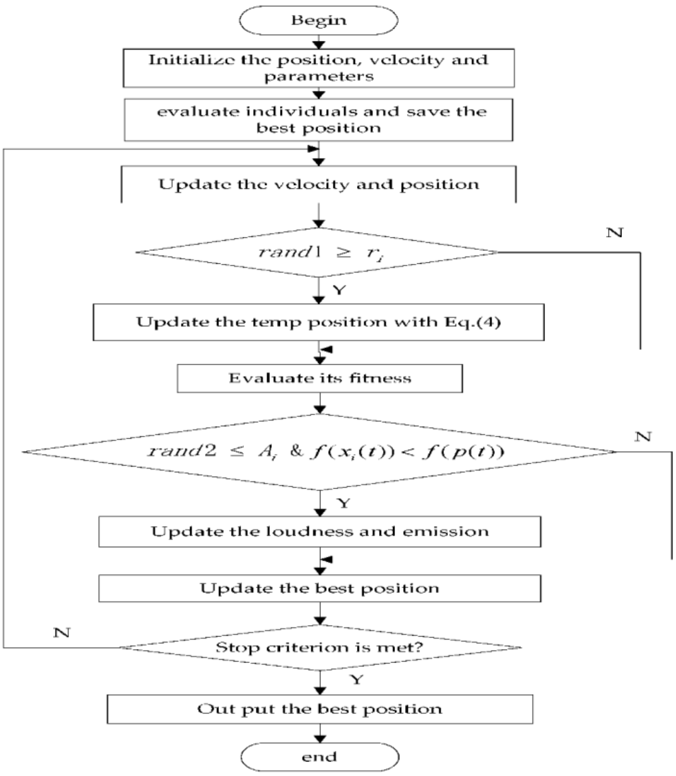

Figure1: Flowchart for BAT algorithm [30]

Therefore, the hybrid bat algorithm with the MLE method was used to estimate the S-S model for the likelihood function. The likelihood function was used as the objective function for the BAT algorithm.

The steps of the BAT-MLE algorithm are presented below:

Step1. For the fitness function of the BAT algorithm, the likelihood function of all solutions in the population was used as : $f = nL_\alpha + nL_\theta + n\ln\beta^{\alpha\theta} + (\alpha\theta - 1)\ln\prod_{i=1}^{n} y_i$, initialize population of BAT x_i, velocity v_i. Determine pulse frequency f_i at x_i. Loudness A and pulse rate r_i are initialized. $r_i \in [0,1]$.

Step2. New solutions are generated and updating positions and velocities by adjusting the frequency.

If (random1 > r_i), select the solution and around the selected best solution generate a neighborhood solution.

Else fly random to create a new solution.

If (random 2 <A_i and (x_i) <$f(x_0)$), whereas $f(x)\,, x = (x_1, \dots x_d)^T$ objective function. Accept the new solution, increase r_i and reduce A_i.

Step3. By ranking the bats, the current best (x_0) was found.

Step4. Post procedure outcomes and representation while (t < maximum number of iteration). The algorithm terminates with the best aggregate solution.

7.Simulation study

In this section, a simulation study was used as the best criterion for investigating the performance between the proposed method (BAT-MLE) and the MLE method. Different sample sizes = (15, 30, 50,75 and 100) based on the MSE criterion with 1000 replicates were used. The steps of the simulation were introduced as follows:

Step1: Generate random samples as $u_i, u_2, \dots, u_n$. and $w_1, w_2, \dots, w_m$, respectively, which follows the continuous uniform distribution specified on the separator timeline (0,1).

Step2: Transform the above uniform random samples to a random sample follows GIPD using the cumulative distribution function as follow:

$F(y)=(\beta y)^{\alpha\theta}$ ➔ $u_i = (\beta y_i)^{\alpha\theta}$ ➔ $y_i = (u_i)^{\frac{1}{\alpha\theta}}/\beta$

Let $\mathcal{M}$ is a vector utilizes for all parameters such as $\mathcal{M}$= [α, β, θ].

Step 3: Calculate the R from Equation (5).

Step 3: Calculate $\hat{R}$ depending on MLE utilizing the Equation (8).

Step 4: Calculate the best solution of ($\hat{R}$) from BAT-MLE algorithm.

Step 5: MSE is calculated as follows for L=1000 iterations: $\mathrm{MSE} = \frac{1}{L}\sum_{i=1}^{L}\left((\hat{R}_i - R)^2\right)$.

Then, calculated relative efficiency (eff) = $\mathrm{MSE}_{\text{BAT-MLE}} / \mathrm{MSE}_{\text{MLE}}$

Random samples for xi and yi were used as n=10, 25, 40, 75 and 100 and yi=10, 25, 40, 75 and 100, respectively.

8.Numerical results

In this section, the simulation results of the proposed estimation algorithm (BAT-MLE) were illustrated in Tables (1-4) and distinguish the following results. Four samples problem size 10, 25, 40, 75 and 100 have been implemented 1000 trials based on two parameters values (θ_i). The simulation study was coded using MATLAB 2021b. According to all tables, it can be seen that the BAT-MLE algorithm of S-S reliability provide less MSE and the estimations are closer to the real parameter values than MLE. Therefore, the BAT-MLE method considered as an effective parameter estimation method for of S-S reliability

Table 1: Reliability, estimation of R, MSE values, when $\alpha = 2$, $\beta = 0.5$, $\theta_1 = 1$ and $\theta_2 = 1.5$.

n	m	R	$\hat{R}_{BAT-MLE}$	$\hat{R}_{MLE}$	eff
	10	0.003844	0.00367	0.006665	2.64E+02
issa	25	0.028741	0.030237	0.044299	1.08E+02
10	40	0.113456	0.126447	0.156808	1.11E+01
	75	0.156787	0.176379	0.209534	7.24E+00
	100	0.217128	0.245665	0.279212	4.73E+00
	10	0.24948	0.282517	0.315062	3.94E+00
	25	0.273896	0.310144	0.341504	3.48E+00
25	40	0.28043	0.317507	0.348497	3.37E+00
	75	0.306212	0.346427	0.375759	2.99E+00
	100	0.397362	0.44672	0.468357	2.07E+00
	10	0.416429	0.467275	0.487061	1.93E+00
	25	0.416467	0.467315	0.487097	1.93E+00
40	40	0.448207	0.501179	0.517774	1.72E+00
	75	0.472153	0.526425	0.54056	1.59E+00
	100	0.48818	0.543172	0.555649	1.51E+00
	10	0.545305	0.601858	0.608448	1.25E+00
	25	0.586573	0.643238	0.645719	1.09E+00
75	40	0.588406	0.645057	0.64736	1.08E+00
	75	0.645466	0.70075	0.697801	8.96E-01
	100	0.653551	0.7085	0.70486	8.72E-01
	10	0.691085	0.744002	0.737366	7.65E-01
	25	0.782268	0.826861	0.814841	5.33E-01
100	40	0.874345	0.905214	0.891767	3.18E-01
	75	0.917407	0.939737	0.927751	2.14E-01
	100	0.990198	0.993703	0.990502	7.54E-03

Table 2: Reliability, estimation of R, MSE values, when $\alpha = 2$, $\beta = 0.5$, $\theta_1 = 0.5$ and $\theta_2 = 2$.

n	m	R	$\hat{R}_{BAT-MLE}$	$\hat{R}_{MLE}$	eff
	10	0.033947	0.03273	0.065992	2.14E+00
	25	0.068888	0.074578	0.085695	6.87E-01
10	40	0.102134	0.117427	0.099383	4.93E-01
	75	0.112261	0.130832	0.103038	4.60E-01
	100	0.113765	0.132834	0.103566	4.56E-01
	10	0.17337	0.213504	0.122186	3.39E-01
	25	0.180578	0.22335	0.124215	3.29E-01
25	40	0.181359	0.224417	0.124433	3.28E-01
	75	0.279835	0.357738	0.149588	2.27E-01
	100	0.30461	0.390427	0.155467	2.07E-01
	10	0.379945	0.486431	0.172937	1.57E-01
	25	0.386525	0.49454	0.174454	1.53E-01
40	40	0.38719	0.495357	0.174607	1.53E-01
	75	0.406638	0.519025	0.179096	1.42E-01
	100	0.468111	0.590904	0.193483	1.11E-01
	10	0.49616	0.622148	0.200225	9.89E-02
	25	0.546107	0.675277	0.212678	7.92E-02
75	40	0.563815	0.693329	0.217271	7.29E-02

	75	0.572089	0.701622	0.219455	7.01E-02
	100	0.605465	0.734149	0.228553	5.94E-02
	10	0.741418	0.851235	0.273039	2.62E-02
	25	0.797925	0.892576	0.297801	1.67E-02
100	40	0.809215	0.900314	0.30351	1.50E-02
	75	0.928963	0.971057	0.400524	2.88E-03
	100	0.935366	0.974199	0.409793	2.48E-03

Table 3: Reliability, estimation of R, MSE values, when $\alpha = 2$, $\beta = 0.5$, $\theta_1 = 1.5$ and $\theta_2 = 0.5$.

n	m	R	$\hat{R}_{BAT-MLE}$	$\hat{R}_{MLE}$	eff
	10	0.051891	0.029981	0.105493	9.86E-01
	25	0.088353	0.056433	0.114957	2.67E+01
10	40	0.149643	0.106455	0.125616	5.54E-01
	75	0.161744	0.117008	0.127335	4.25E-01
	100	0.172268	0.126346	0.128763	3.48E-01
	10	0.173426	0.127383	0.128916	3.41E-01
	25	0.323129	0.273976	0.145335	5.30E-02
25	40	0.328537	0.27967	0.145853	5.00E-02
	75	0.360724	0.314024	0.14888	3.55E-02
	100	0.415281	0.373881	0.153873	1.93E-02
	10	0.415359	0.373967	0.15388	1.93E-02
	25	0.429009	0.389222	0.155115	1.64E-02
40	40	0.55747	0.536549	0.166906	2.40E-03
	75	0.606588	0.593856	0.171708	7.27E-04
	100	0.681895	0.681483	0.179772	5.84E-07
	10	0.68814	0.688696	0.180494	1.04E-06
	25	0.708543	0.712172	0.182928	4.17E-05
75	40	0.714892	0.719444	0.183711	6.40E-05
	75	0.74463	0.753255	0.187565	2.11E-04
	100	0.792397	0.806452	0.194597	4.91E-04
	10	0.809014	0.824548	0.197371	5.72E-04
	25	0.830233	0.847283	0.201244	6.55E-04
100	40	0.866189	0.884666	0.208968	7.07E-04
	75	0.911334	0.929007	0.222175	5.92E-04
	100	0.938971	0.954243	0.234177	4.24E-04

Table 4: Reliability, estimation of R, MSE values, when $\alpha = 2$, $\beta = 0.5$, $\theta_1 = 2$ and $\theta_2 = 3$.

n	m	R	$\hat{R}_{BAT-MLE}$	$\hat{R}_{MLE}$	eff
	10	0.029627	0.046113	0.133103	2.54E-02
	25	0.035708	0.054659	0.135838	3.58E-02
10	40	0.045245	0.067761	0.139403	5.72E-02
	75	0.061684	0.089647	0.144253	1.15E-01
	100	0.072327	0.103428	0.146836	1.74E-01
	10	0.101624	0.140103	0.152605	5.70E-01
	25	0.245463	0.302711	0.169992	5.75E-01
25	40	0.246264	0.303557	0.170066	5.65E-01
	75	0.271556	0.330029	0.172345	3.47E-01
	100	0.338224	0.397634	0.177894	1.37E-01
	10	0.343306	0.402672	0.178298	1.29E-01
	25	0.441882	0.497681	0.185823	4.75E-02
40	40	0.466837	0.52102	0.187685	3.77E-02
	75	0.53259	0.581391	0.192624	2.06E-02
	100	0.572207	0.617091	0.195678	1.42E-02
	10	0.59317	0.635806	0.197336	1.16E-02
	25	0.709814	0.738312	0.207506	3.22E-03
75	40	0.732252	0.757816	0.20975	2.40E-03
	75	0.732806	0.758297	0.209807	2.38E-03
	100	0.744573	0.768512	0.21104	2.01E-03
	10	0.749915	0.773148	0.211614	1.86E-03
	25	0.852183	0.86207	0.225198	2.49E-04
100	40	0.876139	0.883124	0.229574	1.17E-04
	75	0.882435	0.888686	0.230854	9.21E-05
	100	0.980235	0.978734	0.274169	4.53E-06

9. Conclusions

In this paper, a new algorithm BAT-MLE is recommended to estimate the stress-strength model of the reliability system for the generalized inverse Pareto distribution. In addition, a generalized inverse Pareto distribution with some statistical properties has been introduced. Simulations are used to compare the proposed method (BAT-MLE) with the classical method (MLE). The results show that the BAT-MLE variant estimates the parameters more accurately than the MLE.

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