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Anthropogenic Impacts on Urban River Water Quality for Rating Water Pollution Using an Effective Fuzzy Soft Multi Criteria Group Decision Making Model

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Abstract

This study examines the problems with the assessment of river water quality, which is affected by multiple water quality parameters (WQPs) that have inherent uncertainties and uneven decision-maker judgments. To cope with these uncertainties, we use soft sets, fuzzy sets, and fuzzy soft sets. In particular, we first present the square root mean square (SQRTMS) and a water pollution score (WPS) for fuzzy soft sets and propose a novel water pollution assessment framework, i.e., the fuzzy soft multi-criteria group decision-making model (Ω -model). This model produces Ω score, a weighted water pollution score, to classify pollution levels. The proposed methodology is made on water quality indices of Haora River in Tripura India, which serves not only as an essential drinking water source for Agartala but also plays a critical role in supporting local ecosystems, agriculture, and livelihoods. Over the years, the river has faced increasing pollution from domestic wastewater discharge and agricultural runoff, exacerbating water quality issues. Its deteriorating condition has posed a threat to both aquatic life and public health. Our study involves nine key parameters in seven sampling sites situated along the river, which were sampled in January, February, and March of 2024 with an emphasis on waste discharge points. Finally, the proposed model is compared with existing approaches and the effectiveness and benefits are put forth.

Keywords: Multi-criteria decision-making, Soft set, Fuzzy set, Fuzzy soft set, River water quality, Water pollution, Uncertainty modeling.

1. Introduction

An effective water quality assessment is essential to ensure the sustainability of freshwater resources for which rivers, a primary water source, provide drinking, agriculture, and ecosystems. Multiple WQPs affect river water quality and the problem is complicated by the fact that some of the WQPs have an inherent uncertainty in them which is a result of environmental variability and subjective decision-making perspective. For these uncertain conditions, these traditional water quality assessment methods are not capable of effectively handling these uncertainties, and sophisticated decision-making models need to address these uncertainties. To address these issues, soft set theory (SS-theory), fuzzy sets, and fuzzy soft

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sets (FSSs) have been developed as useful tools for handling imprecise and uncertain information.

In 1999, Molodtsov [1] introduced SS-theory as a mathematical tool to deal with ambiguity, complexity, and uncertainty in multi-criteria decision-making (MCDM). Later, Maji et al. [2] established some novel concepts within SS-theory, such as intersections, unions, complements, and subset operations, and demonstrated how these concepts could be applied to solve real-world decision-making problems (DMPs). In 2002, Maji et al. [3] were the ones who had shown for the first time the practical application of SS-theory in solving DMPs. After that, Ali et al., [4] introduced novel operations in SS-theory and studied their properties and applications in decision-making processes. Alcantud and Santos-García [5] proposed a new criterion for a soft set-based decision-making model to solve real-life DMPs under incomplete information. Chen et al., [6] introduced the generalized vague N-soft set as a generalization of SS-theory and developed a novel group decision-making technique to solve real-life DMPs. Dalkılıç [7] proposed a novel decision-making technique based on SS-theory for decision-making under uncertain conditions. In recent years, uncertainty Modeling has found relevance in epidemiological analysis, and Dalkılıç and Demirtaş [8] further advance that proposition with the development of an SS-theory based algorithm to solve COVID-19 outbreaks.

In real-life decisions, uncertainty plays a significant role in decision-making across many fields, and it is important to have robust mathematical frameworks to handle imprecise information. One of the first contributions in this field was Zadeh's [9] introduction of fuzzy set (FS) theory from which it was possible a structured way to handle vagueness by representing and analyzing data. In the past years, interval valued fuzzy sets, intuitionistic fuzzy sets, hesitant fuzzy sets, and fuzzy multisets, has been emerged as new extensions of fuzzy sets to the purpose of reaching their applicabilities to the most numerous areas of practical fields, medical diagnosis, pattern recognition, in databases and MCDM. Recently, Torkaman and Darvish Falehi [10] also added in their report, by proposing a fractional order interval type 2 fuzzy controller implemented on dynamic voltage restorer based on inverted asymmetric multilevel inverter to enhance power network stability. At the same time, maximizing caste sale with this approach is applied [11], which they used to neural fuzzy sets theory in the framework of MCDM.

The FSS [12], introduced by Maji et al., is a very significant extension in this field as a combination of soft set and fuzzy set with the ability to determine the decision under uncertainty. An FSS based decision-making framework was further developed by Roy and Maji [13] that helps to pave the way for subsequent research. Feng et al., [14] noted that the need for adapting decision-making procedures that can deal with various levels of uncertainty and decision-maker preferences exists. Recently, Paik and Mondal [15] suggested a distance similarity model that combines FS and FSS and thus establishes a unified model for solving decision-making problems under conditions of uncertainty. Gao and Wu [16] developed the idea of filters and examined their fundamental properties and utilizations in fuzzy soft topological spaces. Later, Bhardwaj and Sharma [17] presented an innovative uncertainty measure using fuzzy soft sets and demonstrated its utilization in real-life DMPs. Recently; Das et al., [18] described several fundamental operations on FSSs and elaborated further their properties and utilizations in MCDM methods. Liu et al., [19] established an expert system utilizing FSSs for COVID-19 prediction and diagnosis. Furthermore, Wen [20] introduced the idea of weighted hesitant fuzzy SS-theory, enhancing MCDM frameworks by effectively handling uncertainty and hesitation in assessments.

The Water Quality Index (WQI) is one of the prominent models known for its accuracy and utility in evaluating surface water quality (WQ). Several models of the WQI are available to evaluate the consistency of river WQ. In 1965, Horton introduced the first WQI model, based on ten WQPs which are considered important in the majority of water bodies [21]. Later, Brown [22] created the NSF-WQI, a more stringent version of Horton's WQI technique to provide a more standardized approach to assessing water quality based on multiple WQPs. Since then, many researchers and agencies have attempted to create and improve WQIs for evaluating water quality (see [23]). Kumar et al., [24] analyzed the effects of climate change on the upper Kharun catchment WQ in India, and Guettaf et al., [25] performed a comprehensive evaluation of the Seybouse River's runoff in Algeria. Miller & Hutchins [26] provided an extensive review of the effects of urbanization and climate change on urban flooding and WQ in the UK. Similarly, Shah and Joshi [27] also proposed an innovative WQI model to assess the Sabarmati River in India. The upper Thames River basin was studied by Hutchins et al., [28] regarding combined effects of land use changes and climate stressors on the surface and groundwater quality. Key drivers identified as responsible for water quality trends in European river basins are suggested by Diamantini et al., [29]. Rao et al., [30] presented a trend analysis of the water quality in the Min River sea-entry section in China. Studies of recent times have highlighted the role played by technological advancements in WQI based assessments. The paper by Patel et al., [31] addressed urbanization and industrialization on the Sabarmati River in Ahmedabad and argued for the necessity of monitoring in urban regions. In Ahmed et al., [32], machine learning techniques were used to increase the efficacy of WQI based pollution assessment particularly in Rawal Dam, Pakistan. They demonstrated how combining traditional WQI with advanced machine learning techniques can enhance the accuracy of pollution assessment. Furthermore, Patel and Chitnis [33] introduced an innovative river WQI model using fuzzy logic to investigate the effects of climate alteration and industrialization on the Sabarmati River, India.

In this paper, we present the SQRTMS operation and a WPS for FSSs to enhance water quality assessment. The SQRTMS operation is used in the proposed model to aggregate fuzzy membership values, providing a robust mechanism for handling uncertainties in water quality parameters and allowing for a more accurate assessment of water quality. We demonstrate how effective these methods are by applying them to the Haora River in India, which is an important drinking water source for Agartala and plays a key role in supporting local ecosystems and agriculture. Spanning 47.21 km from the Baramura range to the Titas River in Bangladesh, the Haora River has experienced significant water quality deterioration. Sampling at seven strategic sites from January to March 2024, our findings reveal the river's deteriorated water quality, rendering the river unfit for human consumption. Comparative analyses with existing models validate the advantages of our approach. The structure of the paper is as follows: Section 2 discusses the key theoretical concepts such as soft set, fuzzy set, and fuzzy soft set. Section 3 presents the SQRTMS operation and corresponding WPS for FSS then develops an effective water pollution rating system (Ω model). Section 4 compares the proposed approach against existing techniques by showing its benefits. The study ends finally, in section 5, by summarizing the key findings and providing the guidance for future research.

2. Preliminaries

Let V be a universal set, and let $P(V)$ denote its power set. Consider P to be a nonempty set of parameters that represent the qualities, properties, or features of V 's elements.

Definition 2.1: [1] A soft set over V is defined as an ordered pair (Ψ, P) , where Ψ is a function:

$$\Psi: P \rightarrow P(V).$$

This function associates each attribute $p \in P$ with a subset of V , indicating which components of V satisfy the given parameter. The soft set (Ψ, P) represents uncertain or imprecise information in a flexible and structured manner by associating V elements with certain qualities in P .

Example 2.2: Consider a music streaming service that categorizes songs according to several attributes. Let $V = \{a, b, c, d\}$ represents the four songs offered on the platform. The set of parameters, P , contains of main features needed to classify the songs:

$$P = \{p1 = \text{upbeat}, p2 = \text{instrumental}, p3 = \text{classical}, p4 = \text{trending}\}$$

A soft set (Ψ, P) maps each parameter to a subset of songs with that characteristic:

- $\Psi(p1) = \{a, d\}$ for songs classed as "upbeat"
- $\Psi(p2) = \{b, d\}$ for "instrumental" songs.
- $\Psi(p3) = \{c\}$ for songs in the "classical" genre.
- $\Psi(p4) = \{b, c\}$ for current "trending" tracks.

Thus, the soft set (Ψ, P) is defined as:

$$(\Psi, P) = \{(p1, \{a, d\}), (p2, \{b, d\}), (p3, \{c\}), (p4, \{b, c\})\}.$$

This formulation describes an organized way to categorizing songs based on many features, demonstrating the practical use of soft sets in classification and decision-making settings.

Definition 2.3: [9] A fuzzy set (FS) over a universal set (V) is defined as a collection of ordered pairs $X = \{(o, \mu_X(o)): o \in V\}$ where $\mu_X: V \rightarrow [0,1]$ is a mapping known as the membership function or fuzzy membership function. This function assigns a real number between 0 and 1 to each element o in V , signifying the degree of membership in the fuzzy set X . The closer the membership value $\mu_X(o)$ is to 1, the greater the association of o with the set X , while values approaching 0 suggest a weaker relationship.

Unlike classical sets, which only allow for complete membership (1) or non-membership (0), fuzzy sets allow for partial membership, making them helpful for modeling uncertainty, vagueness, and imprecise information in a wide range of real-world applications.

In this study, we indicate by $FS(V)$ the collection of all possible fuzzy sets defined over V , which represents the sum of all fuzzy classifications that may be applied to the elements of V .

Definition 2.4: [12] A pair (ψ, P) is referred to as a fuzzy soft set (FSS) over V . The mapping $\psi: P \rightarrow FS(V)$ assigns each attribute $p \in P$ to a fuzzy subset of V . In this representation, uncertainty is captured via membership functions μ_ψ . These functions assign a real integer in the interval $[0,1]$ to each element of V , reflecting the degree to which that value fulfils a certain parameter.

Example 2.5: In the context of a music streaming platform as Example 2.2, categorizing songs based on key criteria is frequently imprecise. Instead of binary classification, we propose a fuzzy soft set (Ψ, P) , where each song is assigned a membership value in the interval $[0,1]$ to reflect the degree of a certain attribute.

Let $V = \{a, b, c, d\}$ represent the set of four songs available on the platform, and let $P = \{p1 = \text{upbeat}, p2 = \text{instrumental}, p3 = \text{classical}, p4 = \text{trending}\}$ denote the classification parameters. The fuzzy soft set (Ψ, P) is given by:

$$(\Psi, P) = \left\{ \begin{array}{l} (p1, \{(a, 0.9), (b, 0.3), (c, 0.2), (d, 0.8)\}) \\ (p2, \{(a, 0.4), (b, 0.8), (c, 0.3), (d, 0.7)\}) \\ (p3, \{(a, 0.2), (b, 0.3), (c, 0.9), (d, 0.4)\}) \\ (p4, \{(a, 0.3), (b, 0.7), (c, 0.8), (d, 0.5)\}) \end{array} \right\}$$

- Song a has a high membership value (0.9) in the "upbeat" category, indicating a strong fit.
 - Song b is mostly "instrumental" (0.8) with a moderate amount of "trending" (0.7).
 - Song c has high "classical" (0.9) and moderate "trending" (0.8) scores, but lower values in other categories.
 - Song d is categorized as "upbeat" (0.8) and "instrumental" (0.7), indicating its versatility.
- This fuzzy soft set format enables a more sophisticated classification of music, capturing partial membership in various categories rather than a strict binary assignment.

3. Fuzzy soft multi-criteria group decision-making model

This section introduces the SQRTMS operation (Θ) and the WPS (Ω) developed for fuzzy soft sets. The Ω -model, developed using unique mathematical structures, provides an effective framework for assessing water pollution.

Definition 3.1: Let (φ, P) and (ψ, P) be two FSSs constructed over a non-empty universe V , where each parameter $pt_k \in P$ is associated with membership values in the range $[0,1]$. The SQRTMS operation between (φ, P) and (ψ, P) is represented as follows:

$$(\varphi, P) \Theta (\psi, P) = (\Psi, P).$$

The membership function of the fuzzy soft set (Ψ, P) is determined by the following equation:

$$\forall ss_i \in V, \forall pt_k \in P, \mu_{\Psi(pt_k)}(ss_i) = \left(\frac{1}{2} (\mu_{\varphi(pt_k)}^2(ss_i) + \mu_{\psi(pt_k)}^2(ss_i)) \right)^{\frac{1}{2}}.$$

Let $(\varphi_1, P), (\varphi_2, P), \dots, (\varphi_q, P) \in FSS(V)$. Then the SQRTMS of $(\varphi_1, P), (\varphi_2, P), \dots, (\varphi_q, P)$ denoted by $(\varphi_1, P) \Theta (\varphi_2, P) \Theta \dots \Theta (\varphi_q, P)$ and defined as a FSS (Ψ, P) , where $\forall ss_i \in V, \forall pt_k \in P$,

$$\mu_{\Psi(pt_k)}(ss_i) = \left(\frac{1}{q} (\mu_{\varphi_1(pt_k)}^2(ss_i) + \mu_{\varphi_2(pt_k)}^2(ss_i) + \dots + \mu_{\varphi_q(pt_k)}^2(ss_i)) \right)^{\frac{1}{2}}.$$

The SQRTMS operation creates a combined membership function by taking the square root of the mean of the squared membership values of two FSSs. This method effectively captures the influence of both input FSSs while minimizing severe changes, resulting in a balanced aggregation of uncertainty.

Definition 3.2: Consider the FSS (Φ, P) defined over a non-empty universe (V) . Assume each parameter $pt_k \in P$ is connected with a weight ϖ_k , where the weight function $\varpi: P \rightarrow [0,1]$ assigns a relevance level to each parameter. Then the WPS for each sampling station s in V is defined as

$$\Omega_{\varpi}(s) = \frac{\sum_{k=1}^m [\varpi(\beta_k) \times \mu_{\Phi(\beta_k)}(s)]}{\sum_{k=1}^m \varpi(\beta_k)}, \beta_k \in P \text{ and } \forall s \in V.$$

3.1 Algorithm (Ω -model)

Step1. Specify the set of sampling stations $S = \{\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n\}$, a WQP set $P = \{\beta_1, \beta_2, \beta_3, \dots, \beta_m\}$, and a group of experts $E = \{e_1, e_2, e_3, \dots, e_q\}$.

Step2. Input the resulting crisp data (water quality data) $DT = \{dt_1, dt_2, dt_3, \dots, dt_q\}$ for all the specified sampling stations $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n$ as provided by each expert ex_k .

Step3. Obtain the recommended ideal and standard values for each WQP $\beta_k \in P$ from environmental agencies or regulatory bodies.

Step4. Construct fuzzy membership functions $\mu_{\beta_k}: R \rightarrow [0,1]$ for each WQP $\beta_k \in P$ based on the ideal and standard values.

Step5. Apply the membership functions to transform the crisp data summaries $dt_1, dt_2, dt_3, \dots, dt_q$ into corresponding fuzzy soft sets $(\psi_1, P), (\psi_2, P), (\psi_3, P)$.

Step6. Compute the resultant FSS $(\Psi, P) = (\psi_1, P) \theta (\psi_2, P) \theta (\psi_3, P) \theta \dots \theta (\psi_q, P)$.

Step7. Assign weight ϖ_k for each WQP β_k in set P .

Step8. Compute the WPS using the formula

$$\Omega_{\varpi}(\alpha_i) = \frac{\sum_{k=1}^m [\varpi(\beta_k) \times \Omega(\mu_{\Psi(\beta_k)}(\alpha_i))]}{\sum_{k=1}^m \varpi(\beta_k)}, \beta_k \in P \text{ and } \forall \alpha_i \text{ in } S.$$

Step9. If WPS $\Omega_{\varpi}(\alpha_k)$ nears zero signifies excellent water quality, while nearing one indicates poor quality.

3.2 Assessment of water pollution of Haora River

This section evaluates water contamination levels in the Haora River, Tripura, India, using the suggested Ω -model. The WPS is a key indication of water quality. WPS around zero indicate great water quality, whereas WPS near one indicate severe contamination. The Ω -model weights numerous water quality parameters (WQPs) to create a composite pollution index that accurately reflects the overall water quality state. To enable a thorough examination, seven strategic sampling sites were carefully selected along the Haora River, ranging from Champaknagar (23.8020°N, 91.4880°E) to Rajnagar (23.8270°N, 91.2570°E). These locations were chosen based on the spatial distribution of garbage release points, such that significant pollution sources were sufficiently represented. Table 1 lists the sampling sites' locations, coordinates, and pollution sources, whereas Figure 1 depicts their geographic dispersion. The WPS computations provide insights into the Haora River's pollution status, aiding effective water quality management and pollution reduction measures.

Table 1 Sampling stations (S).

S	Sampling stations	Location	Coordinates
α_1	Champaknagar, Tripura, India	Bathing ghat	23.802°N, 91.488°E
α_2	Mohanpur, Tripura, India	Bathing ghat	23.817°N, 91.393°E
α_3	Chandrapur, Tripura, India	Bathing ghat	23.836°N, 91.309°E
α_4	Aralia, Tripura, India	Intake point	23.829°N, 91.303°E
α_5	Gandhighat, Tripura, India	Intake point	23.826°N, 91.272°E
α_6	Dashamighat, Tripura, India	Bathing ghat & Immersion ghat	23.827°N, 91.272°E
α_7	Rajnagar (Indo-Bangla border area)	Downstream to Bangladesh	23.827°N, 91.257°E

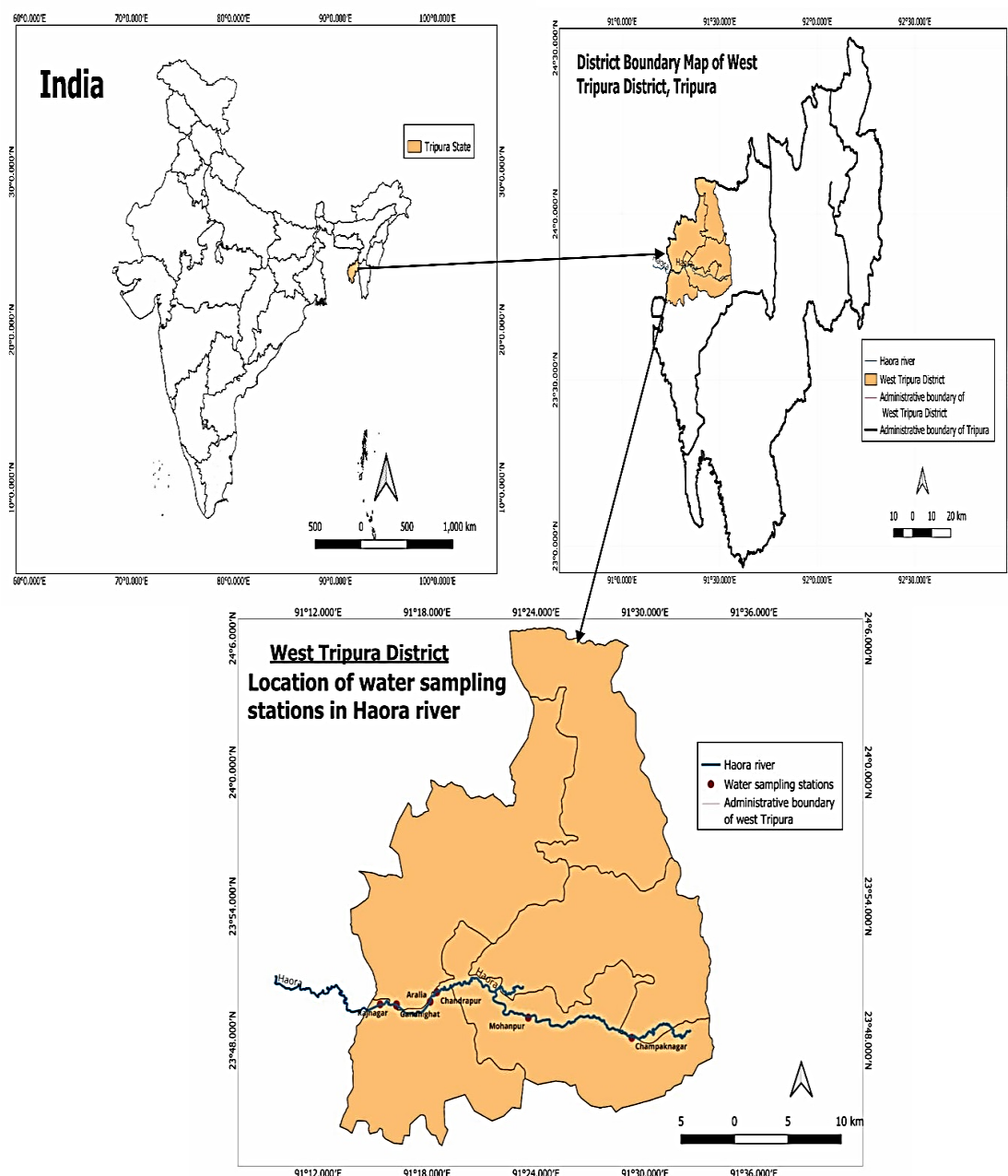


Figure 1: Geographical location and sampling sites along the Haora River.

This figure (Figure 1) illustrates the map of the Haora River in Tripura, highlighting the seven selected sampling sites from Champaknagar (23.802°N , 91.488°E) to Rajnagar (23.827°N , 91.257°E). The locations were strategically chosen to represent variations in water quality influenced by different point sources of waste discharge. The map also displays the surrounding areas and major waste discharge points contributing to the river's pollution, providing a visual context for the study's sampling strategy.

Water samples were collected from seven selected sampling points along the Haora River using a Garmin eTrex Vista Cx GPS gadget from Taiwan, which accurately recorded geographical coordinates (longitude and latitude). Sampling took place across three months, from January to March 2024, with duplicate samples collected from each location to assure data dependability. The spatial placement of sampling points was carefully considered to

capture fluctuations in water quality and account for the blending of discharged waste and river water.

To ensure sample integrity, 1.5-liter polypropylene bottles were employed for collection. To avoid contamination, these bottles were carefully cleansed with 10% HCl in the laboratory before being rinsed with water samples from their various collecting sites. This stringent method guaranteed the accuracy of the measured parameters.

In this study, nine essential WQPs were chosen for analysis, each representing a significant indicator of river health:

- pH (β_1) – Indicator of acidity/alkalinity balance.
- Electrical Conductivity (β_2) – Measures the water's ability to conduct electricity, influenced by dissolved ions.
- Total Suspended Solids (β_3) – Reflects particulate matter in the water.
- Dissolved Oxygen (β_4) – Essential for aquatic life, indicating oxygen availability.
- Biochemical Oxygen Demand (β_5) – Measures organic pollution based on microbial oxygen consumption.
- Chemical Oxygen Demand (β_6) – Assesses the total quantity of oxygen required for chemical oxidation of pollutants.
- Hardness (β_7) – Represents dissolved calcium and magnesium content.
- Alkalinity (β_8) – Indicates the water's capacity to neutralize acids.
- Total Dissolved Solids (β_9) – Measures dissolved salts and organic substances in water.

To ensure uniformity and accuracy, the sampling and analytical techniques followed the standards established by the American Public Health Association (APHA) [34]. Tables 2, 3, and 4 summarize the measured WQP values for the seven selected locations in January, February, and March of 2024. To determine water quality compliance, these values were compared to regulatory standards established by:

- Bureau of Indian Standards (BIS) [35].
- Indian Council for Medical Research (ICMR) [36].
- World Health Organization (WHO) [37].

The comparison study, provided in Table 5, serves as a benchmark for assessing the level of pollution in the Haora River and establishing its acceptability for various applications, such as drinking, agricultural, and aquatic sustainability.

Table 2 Water quality data for January 2024.

S	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9
α_1	6.67	176	99.55	5.65	5.45	5.96	109.67	134.35	176
α_2	6.76	185	93.65	5.98	6.44	7.86	89.76	137.57	163
α_3	6.85	189	87.50	6.05	7.75	8.09	95.45	123.76	154
α_4	6.59	197	85.65	5.35	8.04	8.71	100.56	128.89	185
α_5	6.56	186	95.45	4.89	8.68	9.36	97.75	146.47	181
α_6	6.45	164	91.05	5.25	10.12	11.23	91.98	164.43	169
α_7	6.25	184	98.45	4.35	10.55	11.66	101.87	179.59	179

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Table 3: Water quality data for February 2024.

S	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9
α_1	6.85	187	92.59	6.75	3.77	06.62	96.69	121.93	173
α_2	6.76	199	75.18	6.67	7.01	07.99	76.16	120.51	179
α_3	6.65	203	82.28	6.69	5.07	04.23	78.81	116.22	152
α_4	6.54	208	96.41	6.43	5.27	05.23	81.45	107.22	149
α_5	6.58	198	90.74	5.52	8.31	10.21	79.71	107.65	144
α_6	6.43	178	75.29	5.42	8.52	11.58	70.23	109.79	151
α_7	6.21	189	94.23	5.81	9.05	12.15	76.78	101.46	143

Table 4: Water quality data for March 2024.

S	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9
α_1	6.61	190.31	81.11	6.37	4.46	7.81	85.64	163.52	172.04
α_2	6.71	203.98	71.29	5.72	6.54	7.71	75.71	116.27	180.11
α_3	6.68	183.87	77.72	6.57	7.68	8.46	65.12	97.65	156.72
α_4	6.47	172.71	95.05	6.18	7.97	8.88	55.39	88.02	157.07
α_5	6.39	174.98	89.64	4.89	7.89	9.45	75.89	86.27	159.74
α_6	6.49	178.67	76.91	5.56	8.89	9.65	63.36	90.77	158.22
α_7	6.37	170.87	92.46	4.94	9.84	10.05	59.76	86.9	130.52

Table 5 : Ideal and standard values of the WQPs.

P	Parameter name	Standard value	Ideal value	Recommending agencies
β_1	pH	6.5-8.5	7	ICMR [36]/BIS [35]
β_2	Electrical Conductivity (mho/cm)	300	0	ICMR [36]
β_3	Total Suspended Solids (mg/l)	500	0	WHO [37]
β_4	Dissolved Oxygen (mg/l)	5.00	14.6	ICMR [36]/WHO [37]
β_5	Biological Oxygen Demand (mg/l)	5.00	0	ICMR [36]
β_6	Chemical Oxygen Demand (mg/l)	10	0	WHO [37]
β_7	Total Hardness(mg/l)	300	0	ICMR [36]/BIS [35]
β_8	Total Alkalinity (mg/l)	200	0	BIS [35]
β_9	Total Dissolved Solids (mg/l)	500	0	ICMR [36]/BIS [35]

Table 6 :Fuzzy membership functions for different WQPs.

P	Parameters	Fuzzy membership functions
β_1	pH	$\mu_{\beta_1}(x) = \begin{cases} \frac{7.0 - x}{7.0 - 6.5}; & \text{when } 6.5 \leq x \leq 7.0 \\ \frac{x - 7.0}{8.5 - 7.0}; & \text{when } 7.0 \leq x \leq 8.5 \\ 1; & \text{otherwise} \end{cases}$
β_2	Electrical conductivity	$\mu_{\beta_2}(x) = \begin{cases} \frac{x - 0}{300 - 0}; & \text{when } 0 \leq x \leq 300 \\ 1; & \text{when } x \geq 300 \end{cases}$
β_3	Total suspended solid	$\mu_{\beta_3}(x) = \begin{cases} \frac{x - 0}{500 - 0}; & \text{when } 0 \leq x \leq 500 \\ 1; & \text{when } x \geq 500 \end{cases}$
β_4	Dissolve oxygen	$\mu_{\beta_4}(x) = \begin{cases} \frac{14.6 - x}{14.6 - 5}; & \text{when } 5 \leq x \leq 14.6 \\ 1; & \text{when } x \leq 5 \\ 0; & \text{when } x \geq 14.6 \end{cases}$
β_5	Biochemical oxygen demand	$\mu_{\beta_5}(x) = \begin{cases} \frac{x - 0}{5 - 0}; & \text{when } 0 \leq x \leq 5 \\ 1; & \text{when } x \geq 5 \end{cases}$
β_6	Chemical oxygen demand	$\mu_{\beta_6}(x) = \begin{cases} \frac{x - 0}{10 - 0}; & \text{when } 0 \leq x \leq 10 \\ 1; & \text{when } x \geq 10 \end{cases}$
β_7	Hardness	$\mu_{\beta_7}(x) = \begin{cases} \frac{x - 0}{300 - 0}; & \text{when } 0 \leq x \leq 300 \\ 1; & \text{when } x \geq 300 \end{cases}$
β_8	Alkalinity	$\mu_{\beta_8}(x) = \begin{cases} \frac{x - 0}{200 - 0}; & \text{when } 0 \leq x \leq 200 \\ 1; & \text{when } x \geq 200 \end{cases}$
β_9	Total dissolved solid	$\mu_{\beta_9}(x) = \begin{cases} \frac{x - 0}{500 - 0}; & \text{when } 0 \leq x \leq 500 \\ 1; & \text{when } x \geq 500 \end{cases}$

The steps below detail the technique for analyzing the water quality of the Haora River using FSSs and the SQRMS operation (Θ).

Step 1: Let $S = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7\}$ (Table 2 for locations and coordinates) denote the collection of sample sites along the Haora River and $P = \{\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7, \beta_8, \beta_9\}$ as the set of WQPs being considered. Each β_k indicates a crucial water quality indicator.

Step 2: Collect water quality data summaries of WQPs across three seasons:

- Pre-monsoon (Table 2).
- Monsoon (Table 3).
- Post-monsoon (Table 4).

These data represent the measured values from the selected sample stations during the study period.

Step 3: Establish Ideal and Standard Values

- Refer to Table 5 for the ideal and standard values of WQPs based on regulatory guidelines (BIS [35], ICMR [36], WHO [37]).
- These values serve as reference points for determining the level of pollution in the river.

Step 4: Using the ideal and standard values from Table 5, construct fuzzy membership functions for each WQP. The fuzzy membership functions provide a normalized representation of pollution levels, converting crisp values into fuzzy values within the interval [0,1]. The detailed definitions of these functions are presented in Table 6.

Step 5: Transform the crisp WQP data from Tables 2, 3, and 4 into fuzzy soft sets for each season:

- Pre-monsoon: (ψ, P) (Table 7).
- Monsoon: (ϕ, P) (Table 8).
- Post-monsoon: (σ, P) (Table 9).

These fuzzy soft sets express the pollution levels in a more adaptive and uncertainty-aware manner.

Step 6: Apply the SQRMS operation Θ to aggregate the seasonal FSSs into a single resultant FSS $(\Psi, P) = (\psi, P)\Theta(\phi, P)\Theta(\sigma, P)$. This step ensures a comprehensive representation of water quality by integrating data across all three seasons. The final aggregated fuzzy soft set is presented in Table 10.

Table 7: The FSS (ψ, P) .

S	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9
α_1	0.66	0.587	0.199	0.932	1	0.596	0.366	0.672	0.352
α_2	0.48	0.617	0.187	0.898	1	0.786	0.299	0.688	0.326
α_3	0.30	0.630	0.175	0.891	1	0.809	0.318	0.619	0.308
α_4	0.82	0.657	0.171	0.964	1	0.871	0.335	0.644	0.370
α_5	0.88	0.620	0.191	1	1	0.936	0.326	0.732	0.362
α_6	1	0.547	0.182	0.974	1	1	0.307	0.822	0.338
α_7	1	0.613	0.197	1	1	1	0.340	0.898	0.358

Table 8 :The FSS (ϕ, P) .

S	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9
α_1	0.30	0.623	0.185	0.818	0.754	0.662	0.322	0.610	0.346
α_2	0.48	0.663	0.150	0.826	1	0.799	0.254	0.603	0.358
α_3	0.70	0.677	0.165	0.824	1	0.423	0.263	0.581	0.304
α_4	0.92	0.693	0.193	0.851	1	0.523	0.272	0.536	0.298
α_5	0.84	0.660	0.181	0.946	1	1	0.266	0.538	0.288
α_6	1	0.593	0.151	0.956	1	1	0.234	0.549	0.302
α_7	1	0.630	0.188	0.916	1	1	0.256	0.507	0.286

Table 9: The FSS (σ, P) .

S	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9
α_1	0.78	0.634	0.162	0.857	0.892	0.781	0.285	0.818	0.344
α_2	0.58	0.680	0.143	0.925	1	0.771	0.252	0.581	0.360
α_3	0.64	0.613	0.155	0.836	1	0.846	0.217	0.488	0.313
α_4	1	0.576	0.190	0.877	1	0.888	0.185	0.440	0.314
α_5	1	0.583	0.179	1	1	0.945	0.253	0.431	0.320
α_6	1	0.596	0.154	0.942	1	0.965	0.211	0.458	0.316
α_7	1	0.570	0.185	1	1	1	0.199	0.435	0.261

Table 10: The FSS $(\Psi, P) = (\psi, P)\theta(\varphi, P)\theta(\sigma, P)$.

S	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9
α_1	0.614	0.615	0.183	0.870	0.89	0.684	0.326	0.705	0.347
α_2	0.515	0.654	0.161	0.884	1	0.785	0.269	0.626	0.348
α_3	0.574	0.640	0.165	0.851	1	0.719	0.269	0.565	0.308
α_4	0.916	0.644	0.185	0.899	1	0.779	0.271	0.547	0.329
α_5	0.909	0.622	0.184	0.982	1	0.961	0.283	0.581	0.325
α_6	1	0.579	0.163	0.957	1	0.988	0.254	0.628	0.319
α_7	1	0.605	0.190	0.973	1	1	0.271	0.646	0.304

Step7. To determine relative weight α_i to each WQP p_i in set P, we use the following formula:

$$\alpha_i = \frac{1}{S_i}$$

where S_i denotes the recommended standard value for the i th WQP $p_i \in P$. Using this equation, we get the relative weights of the WQPs as follows:

$\alpha = \{\alpha_1 = 0.118; \alpha_2 = 0.003; \alpha_3 = 0.002; \alpha_4 = 0.2; \alpha_5 = 0.2; \alpha_6 = 0.1; \alpha_7 = 0.003; \alpha_8 = 0.005; \alpha_9 = 0.002\}$.

Step8. Using the aggregated fuzzy soft set (Ψ, P) from Step 6 (Table 10) and the relative weights α from Step 7, we computed the WPS for each sample station along the Haora River. The resulting values are:

Table 10: WPS for each sample station along the Haora River.

S	Sampling Station	WPS
α_1	Champaknagar	0.790
α_2	Mohanpur	0.826
α_3	Chandrapur	0.816
α_4	Aralia	0.904
α_5	Gandhighat	0.958
α_6	Dashamighat	0.971
α_7	Rajnagar	0.979

These values, which range from 0.79 to 0.979, show that water quality is low at all stations. Champaknagar has the lowest WPS (0.79), indicating better water quality than other locations. Rajnagar has the highest WPS (0.979), indicating the worst water quality. The findings confirm considerable pollution in the Haora River, rendering the water unsafe for human consumption and household use. The decline in water quality is mostly due to domestic wastewater flow from nearby residential and industrial sectors. Given the high pollution levels, quick action is required to implement wastewater treatment methods,

enhance pollution control policies, and increase community awareness to protect the river and its dependent communities. Table 12 shows the ranking order of sampling sites based on WPSs, indicating the severity of pollution in the river.

4. Comparison analysis

The effectiveness of our suggested Ω -model is evaluated by comparing its performance to established water quality assessment models. The section provides a thorough examination of the contrasts between our approach and traditional methods, with a focus on its superior capabilities in dealing with uncertainty and complex decision-making scenarios.

4.1 Comparison with the Brown WQI

The Brown WQI model [22] functions as a popular assessment tool that derives WQI values through weighted arithmetic mean operations from predetermined weighting criteria for different WQPs. The WQI values computed through our analysis of crisp data summaries via this model established river water quality as unsuitable for drinking purposes because they spanned from 76 to 100 at all sampling locations along the Haora River. The computed results from both the Brown WQI model and our proposed model showed accordant rankings for the sampling locations (Table 12) with Champaknagar appearing as the least afflicted site and Rajnagar as the most contaminated area. The calculated scores exhibit significant distinctions between them. The Brown WQI model utilizes weighted arithmetic WQI values but our model implements new calculation procedures based on weighted arithmetic and weighted geometric. The assessment becomes more advanced through the incorporation of ideal and standard WQP values which produce refined scores.

The Brown WQI model functions with numerical data only so it does not provide any mechanism to handle uncertain situations. Our methodology integrates FSSs with the SQRTMS operation to analyze uncertain and imprecise water quality data effectively thus making it adaptable for dynamic environments impacted by environmental and human-related factors. Our decision-making model resolves complex DMPs when applied to MCDM challenges which surpasses the classification function of the Brown WQI model.

4.2 Comparison with the CCME WQI model

The Canadian Council of Ministers of the Environment (CCME) WQI model [23] serves as one of the widely used assessment methods for evaluating water quality. The model's application to WQP crisp data summaries produced WQI values ranging from 45 to 65, placing the water quality ratings of the Haora River in the "marginal" category across all sampling sites. The ranking system of both models matched at all sampling stations (Table 12). The CCME WQI model fails to consider weightage between parameters since it treats all Water Quality Parameters equally important. Our model distributes weight values among different parameters through an assigned weighting system that provides crucial pollutants greater emphasis in the total assessment process. By including both ideal and standard WQP values, our model delivers a more exact and comprehensive assessment of water quality. Additionally, our model can solve real-life MCDM problems based on FSSs, which the CCME WQI model cannot.

4.3 Comparison with the Fuzzy WQI model

Fuzzy WQI assessment as per the Patel model represents an improved method over traditional methods because of its enhanced adaptability for water quality measurement [33]. Results from the Patel model based on fuzzy data summaries of WQPs (Tables 7, 8, and 9) generated an average fuzzy WQI scale from 0.75 to 0.95 as a way to show poor water quality

at all sampling stations. The ranking system provided by the Patel model identical to our model displayed the stations in the same sequence (Table 12). The main distinction exists in the approach used for calculation. The Patel model utilizes average fuzzy WQI values for computation while our model incorporates scores that result from the SQRTMS calculation process. Using the SQRTMS operator stabilizes results while achieving feasibility through the reduction of variability that affects basic averaging procedures. The Patel model provides fuzzy logic implementation yet fails to provide suitable solutions for fuzzy soft systems when approaching real MCDM challenges. The purpose of our model is to handle decisions under uncertain conditions so it offers superior capabilities when managing water resources in practice.

In summary, our research yields promising outcomes compared to present methods, particularly due to the novel SQRTMS operator in our suggested model, which enhances stability and practicality. The flexibility of using a WPS rather than a simple score value makes our model better suited for various real-life applications. Our research has successfully bridged existing gaps while making valuable contributions but more exploration along with refinement is needed for achieving a full and comprehensive strategy.

Table 12: Comparison of water quality assessment models for pollution rating.

Models	Best optimal choice	Ranking according to good quality	quality
Brown WQI model [22]	α_1	$\alpha_1 \gg \alpha_3 \gg \alpha_2 \gg \alpha_4 \gg \alpha_5 \gg \alpha_6 \gg \alpha_7$	Very poor
CCME WQI model [23]	α_1	$\alpha_1 \gg \alpha_3 \gg \alpha_2 \gg \alpha_4 \gg \alpha_5 \gg \alpha_6 \gg \alpha_7$	Marginal
Patel-model [33]	α_1	$\alpha_1 \gg \alpha_3 \gg \alpha_2 \gg \alpha_4 \gg \alpha_5 \gg \alpha_6 \gg \alpha_7$	Poor
Proposed model	α_1	$\alpha_1 \gg \alpha_3 \gg \alpha_2 \gg \alpha_4 \gg \alpha_5 \gg \alpha_6 \gg \alpha_7$	Very Poor

5. Conclusions

The study highlights the deteriorated water quality of the Haora River, which serves not only as an essential drinking water source for Agartala but also for sustaining local ecosystems, agriculture, and the livelihoods of surrounding communities. Using the innovative Ω -model, nine key water quality parameters were analyzed from samples collected between January and March 2024. The results revealed consistently high pollution levels across all stations, primarily due to high biochemical oxygen demand from domestic wastewater dumping. The WPS indicated the river's unsuitability for human use, underscoring the urgent need for regular monitoring, public awareness, and strict enforcement of waste treatment laws to protect the river's health. Our research demonstrates promising outcomes with the SQRTMS and WPS, enhancing the model's stability, practicality, and adaptability for various real-world applications compared to existing models. While we addressed many challenges, further investigation and improvement are needed.

In our future studies, we aim to expand the applications of the fuzzy soft multi-criteria group decision-making model to various environmental contexts. We plan to assess water quality in other urban rivers facing similar pollution challenges, which will provide comparative data and enhance the model's generalizability. Additionally, we will explore the integration of remote sensing technologies to facilitate real-time monitoring of water quality parameters, enabling timely decision-making and improved management strategies. Our research will also investigate interdisciplinary applications of the proposed model in fields such as urban planning, medical diagnostics, and environmental management to address

diverse ecological and health-related issues. Furthermore, we intend to examine additional water quality parameters and socio-economic factors that influence decision-making processes, thereby refining the model's accuracy and applicability. Engaging the public through community involvement and raising awareness about water quality issues will be crucial for enhancing monitoring and management efforts. Finally, we aim to collaborate with policymakers to translate our research findings into actionable frameworks and regulations that can effectively mitigate pollution and protect vital water resources.

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