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Orthogonal Generalized Symmetric Higher bi-Derivations on Semiprime Γ-Rings .

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Abstract

In this paper a Γ-ring M is presented. We will study the concept of orthogonal generalized symmetric higher bi-derivations on Γ-ring. We prove that if M is a 2-torsion free semiprime Γ-ring , D_n and G_n are orthogonal generalized symmetric higher bi-derivations associated with symmetric higher bi-derivations d_n *and* g_n respectively for all n ϵN. Then the following relations are hold for all $x, y, z\epsilon M, \alpha\epsilon\Gamma$ and nϵN:

(i) $D_n(x,y)\alpha G_n(y,z) = G_n(x,y)\alpha D_n(y,z)$ =(0) hence $D_n(x,y)\alpha G_n(y,z)+G_n(x,y)\alpha D_n(y,z) = 0$.

(ii) d_n and G_n are orthogonal and $d_n(x,y)\alpha G_n(y,z)$=$G_n(x,y)\alpha d_n(y,z)$=(0) .

(iii) g_n and $D_n$are orthogonal and $g_n(x,y)\alpha D_n(y,z)$=$D_n(x,y)\alpha g_n(y,z)$=(0) .

(iv) d_n and g_n are orthogonal symmetric higher bi-derivations .

(v) $d_nG_n = G_nd_n = 0$ and $g_nD_n = D_ng_n = 0$.

(vi) $G_nD_n = D_nG_n = 0$.

Keywords: Symmetric Bi-derivations Γ-ring, higher bi-derivations Γ-ring, generalized higher bi-derivations Γ-ring ,orthogonal generalized symmetric higher bi-derivations Γ-ring .

تعامد المشتقات الثنائية المتناظرة على الحلقات شبه الأولية من النمط –Γ تعميم

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الخلاصة

في هذا البحث M هي حلقه من النمط –Γ . سوف ندرس مفهوم تعميم تعامد المشتقات الثنائية المتناظرة على الحلقات شبه اولية من النمط –Γ . سوف نبرهن اذا كانت M حلقة شبه اولية طليقة الالتواء من النمط –Γ وكانت D_n, G_n هما تعميم للمشتقات الثنائية المتناظرة المرتبطة بالمشتقات الثنائية المتناظرة d_n, g_n على التوالي لكل nϵ N . اذأ العلاقات التالية متحققة لكل من αϵΓ , x, y, zϵM :

(i) $D_n(x,y)\alpha G_n(y,z) = G_n(x,y)\alpha D_n(y,z)$ =(0) hence $D_n(x,y)\alpha G_n(y,z)+G_n(x,y)\alpha D_n(y,z) = 0$.

(ii) d_n and G_n are orthogonal and $d_n(x,y)\alpha G_n(y,z)$=$G_n(x,y)\alpha d_n(y,z)$=(0) .

(iii) g_n and $D_n$are orthogonal and $g_n(x,y)\alpha D_n(y,z)$=$D_n(x,y)\alpha g_n(y,z)$=(0) .

(iv) d_n and g_n are orthogonal symmetric higher bi-derivations .

(v) $d_nG_n = G_nd_n = 0$ and $g_nD_n = D_ng_n = 0$.

(vi) $G_nD_n = D_nG_n = 0$.

1. Introducation

Let M and Γ be two additive abelian groups, M is called a Γ ring if the following conditions are satisfied for any $x, y, z\epsilon$M and $\alpha, \beta\epsilon\Gamma$:

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$(i)\ x\alpha y\ \epsilon M$
$(ii)\ x\alpha(y+z) = x\alpha y + x\alpha z$
$x(\alpha+\beta)y = x\alpha y + x\beta y$
$(x+y)\alpha z = x\alpha z + y\alpha z$
$(iii)\ (x\alpha y)\beta z = x\alpha(y\beta z)$

The notion of a Γ-ring was first introduced by **Nobusawa** 1964 [1] and generalized by **Barnes** 1966 [2] as above definition .It is well known that every ring is Γ-ring . M is called prime if xΓMΓy=0 implies that x=0 or y=0 and its said to be $semiprime$ if xΓMΓx=0 implies that x=0 for all x,y ϵ M ,[3] , also M is said to be n-torsion free if nx=0 ,xϵM implies that x=0 where n is positive integer.
In [4**] Jing** defined a derivation on Γ-ring as follows :"An additive mapping d:M→M is said to be derivation on M if d(xαy)=d(x) αy +xα d(y) for all x,y ϵ M and α ϵΓ" .

Sapanci and Nakajima in [5] are defined a Jordan derivation on Γ-ring as follows: "An additive mapping d:M→M is said to be Jordan derivation on Γ-ring if d(xαx) = d(x) αx +xα d(x) for all x ϵ M and α ϵΓ . It is clear that every derivation of a Γ-ring M is Jordan derivation of M".

Ceven and Ozturk in [6] are defined a generalized derivation on Γ-ring as follows :" An additive mapping D:M→M is said to be generalized derivation on M if there exists a derivation d:M→M such that D(xαy)= D(x) αy+ xα d(y) for all x,y ϵ M and α ϵΓ ",also defined a Jordan generalized derivation on Γ-ring as follows:"An additive mapping D:M→M is said to be Jordan generalized derivation if there exists a Jordan derivation d:M→M such that D(xαx) = D(x) αx + xα d(x) for all xϵ M and α ϵΓ .It is clear that every generalized derivation on Γ-ring M is Jordan generalized derivation of M ".

Ashraf and Jamal in [7] are introduced the definition of orthogonal derivation on Γ-ring as follows :" Let d and g be two derivations on M are said to be orthogonal if d(x) ΓMΓ g(y) =(0)= g(y) ΓMΓ d(x) for all x,y ϵM " , also Ashraf and Jamal are defined the orthogonal generalized derivation on Γ-ring as follows :" Let D and G be two generalized derivations on M is said to be orthogonal if D(x) ΓMΓ G(y)=(0)=G(y) ΓMΓ D(x) for all x,y ϵM ".

In [8] **Ozturk et al.** are defined a symmetric bi-derivation on Γ-ring M as follows: "A mapping d:MxM→M is said to be symmetric if d(x,y)=d(y,x) for all x,y ϵM . "A mapping f:M→M defined by f(x)=d(x,x), where d:MxM→M is a symmetric mapping, is called the trace of d and the trace f of d satisfies the relation f(x+y)=f(x)+f(y)+2d(x,y) for all x,y ϵM . A symmetric bi-additive mapping on M×M into M is said to be symmetric bi-derivation on M if d(xαy,z)=d(x,z) αy +xα d(y,z) for all x,y,z ϵ M , α ϵΓ and d is said to be Jordan bi-derivation on M if d(xαx,y)= d(x,y) αx + xα d(x,y) for all x,y ϵ M ,α ϵ Γ " , and authers in [8] introduced the notion of generalized bi- derivation and Jordan generalized bi- derivation on Γ-ring as follows: "A symmetric bi-additive mapping D:MxM→M is said to be generalized bi-derivation if there exists $d: M\times M \to M$ bi-derivation such that D(xαy,z)=D(x,z) αy + xα d(y,z) for all x,y,z ϵ M , αϵ Γ, and D is said to be Jordan generalized bi-derivation if there exists a Jordan bi-derivation d:MxM→M such that D(xαx,y)=D(x,y) αx + xα d(x,y) for all x,y ϵ M ,α ϵ Γ".

Marir and Salih in [9] are introduced the concept of higher bi- derivation on Γ-ring M as follows : " Let D=$(d_i)_{i\in N}$ be a family of bi-additive mapping on on $M\times M\ into\ M$ is said to be higher bi-derivation if d_n(xαy,zαw)=$\sum_{i+j=n} d_i(x,z)\alpha d_j(y,w)$ for all x,y,z,w ϵ M , α ϵΓ " , and D=$(d_i)_{i\in N}$ be a family of bi-additive mapping on MxM into M is said to be Jordan bi-derivation if dn (xαx,yαy)= $\sum_{i+j=n} d_i(x,y)\alpha d_j(x,y)\ for$ all x,y ϵ M , α ϵ Γ , and authers in[9] are defined the generalized higher bi-derivation on Γ-ring M as follows: " Let D=$(D_i)_{i\in N}$ be a family of bi-additive mapping on $M\times M$ into M is said to be generalized higher bi-derivation if there exists a higher bi-derivation $d_n: M\times M \to M$ such that D_n(xαy,zαw)=$\sum_{i+j=n} D_i(x,z)\alpha d_j(y,w)$ for all x,y,z,w ϵ M ,α ϵΓ, and D=$(D_i)_{i\in N}$ be a family of bi-additive mapping on $M\times M\ into\ M$ is said to be Jordan generalized higher bi-derivation if there exists $d_n: M\times M\to M$ a Jordan higher bi-derivation such that D_n(xαx, yαy) $= \sum_{i+j=n} D_i(x,y)\alpha d_j(x,y)\ for$ all x,y ϵ M , α ϵΓ ".

In this paper we will extend of this results to present the concept of orthogonal generalized symmetric higher bi –derivations on *semiprime* Γ-ring, and we proved same of lemmas and theorems about arthogonality .

2. Orthogonal Generalized Symmetric Higher bi-Derivations on *Semiprime* Γ- Rings

In this section we will the definition of orthogonal generalized symmetric higher bi-derivations on a Γ-ring M and we introduced an example and some Lemmas used in our work.
Now, we start with the following definition

Definition (2. 1):
Let D=$(D_i)_{i\in N}$and G=$(G_i)_{i\in N}$ are two generalized symmetric higher bi-derivations on Γ-ring M ,then $D_n$and $G_n$are said to be **orthogonal** if for every x,y,z ϵ M ,n ϵ N :
D_n(x,y) ΓMΓ G_n(y,z) =(0)= G_n(y,z) ΓMΓ D_n(x,y) .
Where
D_n(x,y) ΓMΓ G_n(y,z)=$\sum_{i=1}^{n} D_i\,(x,y)\alpha m\beta G_i(y,z) = 0$
For all m ϵM and α,β ϵΓ .

The following example clarify orthogonal generalized higher bi-derivations on Γ-ring M.

Example (2. 2):
Let d_n and g_n are two symmetric higher bi-derivations on Γ-ring M. Put $M' = M \times M$ and $\Gamma' = \Gamma \times \Gamma$, we define d_n' and g_n' on M' into itself such that $d_n'\big((x,y)\big) = (d_n(x),0)$ and $g_n'\big((x,y)\big) = \big(0, g_n(y)\big)$ for all $(x,y)\epsilon\ M'$ and nϵN . More over if (D_n,d_n) and (G_n,g_n) are generalized symmetric higher bi-derivations on M , we defined D_n' and G_n' on M' into itself such that $D_n'\big((x,y)\big) = (D_n(x),0)$ and $G_n'\big((x,y)\big) = \big(0,G_n(y)\big)$ for all $(x,y)\epsilon M'$ and nϵN .Then (D_n', d_n') and (G_n', g_n') are generalized symmetric higher bi-derivations such that D_n' andG_n' are orthogonal .

Lemma (2. 3): [11]
Let M be a 2-torsion free semiprime Γ-ring and a,b the elements of M. If for all α,β ϵΓ, then the following conditions are equivalent:
(i) $a\alpha M\beta b = 0$
(ii) $b\alpha M\beta a = 0$
(iii) aαMβb+bαMβa=0
(iv) aαMβb+bαMβa=0

If one of these conditions is fulfilled, then $a\alpha b = b\alpha a = 0$.

Lemma (2. 4): [10]
Let M be a 2-torsion free semiprime Γ-ring and a, b the elements of M such that
$a\alpha M\beta b + b\alpha M\beta a = 0$ for all a, βϵΓ ,then $a\alpha M\beta b = b\alpha M\beta a = 0$.

Lemma (2. 5):
Let M be a *semiprime* Γ-ring .Suppose that $D_n$and G_n are bi-additive mappings satisfies D_n(x,y) ΓMΓG_n(x,y)=(0) for all x,yϵM ,nϵN . Then D_n(x,y)ΓMΓG_n(y,z)=(0) for all x,y,z ϵM and nϵN .

Proof:
Suppose that D_n(x, y) ΓMΓ G_n(x,y)=(0)
D_n(x,y) ΓMΓ G_n(x,y) =$\sum_{i=1}^{n} D_i(x,y)\alpha m\beta G_i(x,y) = 0$ (1)
for all α,βϵΓ
Replace x by x+z in (1) for all zϵM we get

$\sum_{i=1}^{n} D_i(x+z,y)\alpha m\beta G_i(x+z,y) = 0$
$\sum_{i=1}^{n}[D_i(x,y) + D_i(z,y)]\alpha m\beta[G_i(x,y) + G_i(z,y)] = 0$
$\sum_{i=1}^{n} D_i(x,y)\alpha m\beta G_i(x,y) + D_i(x,y)\alpha m\beta G_i(z,y) + D_i(z,y)\alpha m\beta G_i(x,y) + D_i(z,y)\alpha m\beta G_i(z,y) = 0$

By equation (1) we get
$\sum_{i=1}^{n} D_i(x,y)\alpha m\beta G_i(z,y) + D_i(z,y)\alpha m\beta G_i(x,y) = 0$
$\sum_{i=1}^{n} D_i(x,y)\alpha m\beta G_i(z,y) = -\sum_{i=1}^{n} D_i(z,y)\alpha m\beta G_i(x,y)$ (2)
Multiplication (2) by $\gamma t\delta \sum_{i=1}^{n} D_i(x,y)\alpha m\beta G_i(z,y)$ for all $t\epsilon M$ and $\gamma,\delta\epsilon\Gamma$ we get
$\sum_{i=1}^{n} D_i(x,y)\alpha m\beta G_i(z,y)\, \gamma t\delta \sum_{i=1}^{n} D_i(x,y)\alpha m\beta G_i(z,y) = 0$

Since M is semiprime we get
$\sum_{i=1}^{n} D_i(x,y)\alpha m\beta G_i(z,y) = 0$ (3)
Replace $G_i(z,y)$ by $G_i(y,z)$ in (3) we get
$\sum_{i=1}^{n} D_i(x,y)\alpha m\beta G_i(y,z) = 0$
Hence $D_n(x,y)\Gamma M\Gamma G_n(y,z)$ = (0)

Lemma (2. 6):

Let M be a 2-torsion free semiprime Γ-ring such that $a\alpha y\beta z = a\beta y\alpha z$, two generalized symmetric higher bi-derivations D_n and G_n associated with two symmetric higher bi-derivations d_n and g_n respectively for all $n\epsilon N$. Then D_n and G_n are orthogonal if and only if $D_n(x,y)\alpha G_n(y,z) + G_n(x,y)\alpha D_n(y,z) = 0$ for all $x,y,z\epsilon M$, $n\epsilon N$ and $\alpha,\beta\epsilon\Gamma$.

Proof:

Suppose that $D_n(x,y)\alpha G_n(y,z) + G_n(x,y)\alpha D_n(y,z) = 0$
$\sum_{i=1}^{n} D_i(x,y)\alpha G_i(y,z) + G_i(x,y)\alpha D_i(y,z) = 0$ (1)
Replace x by $x\beta w$ in (1) for all $w\epsilon M$ we get
$\sum_{i=1}^{n} D_i(x\beta w,y)\alpha G_i(y,z) + G_i(x\beta w,y)D_i(y,z) = 0$
$\sum_{i=1}^{n} D_i(x,y)\beta d_i(w,y)\alpha G_i(y,z) + G_i(x,y)\beta g_i(w,y)\alpha D_i(y,z) = 0$ (2)
Replace $d_i(w,y)$ by $g_i(w,y)$ in (2) we get
$\sum_{i=1}^{n} D_i(x,y)\beta g_i(w,y)\alpha G_i(y,z) + G_i(x,y)\beta g_i(w,y)\alpha D_i(y,z) = 0$

By Lemma (2-4) we get
$\sum_{i=1}^{n} D_i(x,y)\beta g_i(w,y)\alpha G_i(y,z) = \sum_{i=1}^{n} G_i(x,y)\beta g_i(w,y)\alpha D_i(y,z) = 0$ (3)
Replace $g_i(w,y)$ by m in (3) for all $m\epsilon M$ we get
$D_n(x,y)\ \Gamma M\Gamma\ G_n(y,z) = G_n(x,y)\ \Gamma M\Gamma\ D_n(y,z) = (0)$

Thus D_n and G_n are orthogonal
Conversely , suppose that D_n and G_n are orthogonal
$D_n(x,y)\ \Gamma M\Gamma\ G_n(y,z)$=(0)= $G_n(x,y)\ \Gamma M\Gamma\ D_n(y,z)$
$\sum_{i=1}^{n} D_i(x,y)\alpha m\beta G_i(y,z) = 0 = \sum_{i=1}^{n} G_i(x,y)\alpha m\beta D_i(y,z)$
$\sum_{i=1}^{n} D_i(x,y)\alpha m\beta G_i(y,z) + G_i(x,y)\alpha m\beta D_i(y,z) = 0$

By Lemma (2-3) we get
$\sum_{i=1}^{n} D_i(x,y)\alpha G_i(y,z) = \sum_{i=1}^{n} G_i(x,y)\alpha D_i(y,z) = 0$
$\sum_{i=1}^{n} D_i(x,y)\alpha G_i(y,z) + G_i(x,y)\alpha D_i(y,z) = 0$
Hence $D_n(x,y)\alpha G_n(y,z) + G_n(x,y)\alpha D_n(y,z) = 0$

Lemma (2. 7):

Let M be a 2-torsion free semiprime Γ-ring such that $a\alpha y\beta z = a\beta y\alpha z$, two generalized symmetric higher bi-derivations D_n and G_n associated with two symmetric higher bi-derivations d_n and g_n respectively for $n \epsilon N$.Then D_n and G_n are orthogonal if and only if $D_n(x,y)\alpha G_n(y,z) = 0$ or $G_n(x,y)\alpha D_n(y,z) = 0$ for all $x,y,z\epsilon M$, $n\epsilon N$ and $\alpha,\beta\epsilon\Gamma$.

Proof:

Suppose that $D_n(x,y)\alpha G_n(y,z) = 0$

$D_n(x,y)\alpha G_n(y,z) = \sum_{i=1}^{n} D_i(x,y)\alpha G_i(y,z) = 0$ (1)

Replace x by xβw in (1) for all w∈M we get

$\sum_{i=1}^{n} D_i(x\beta w,y)\alpha G_i(y,z) = 0$

$\sum_{i=1}^{n} D_i(x,y)\beta d_i(w,y)\alpha G_i(y,z) = 0$ (2)

Replace $d_i(w,y)$ by m for all m∈M we get

$\sum_{i=1}^{n} D_i(x,y)\beta m\alpha G_i(y,z) = 0$

Hence we get the require result.

Similarly way if $G_n(x,y)\alpha D_n(y,z) = 0$ we get D_n and G_n are orthogonal .

Conversely , suppose that D_n and G_n are orthogonal

$D_n(x,y)\Gamma M\Gamma G_n(y,z) = (0)$

$\sum_{i=1}^{n} D_i(x,y)\alpha m\beta G_i(y,z) = 0$

By Lemma (2-3) we get

$\sum_{i=1}^{n} D_i(x,y)\alpha G_i(y,z) = 0$

Hence $D_n(x,y)\alpha G_n(y,z) = 0$

And by $G_n(x,y)\Gamma M\Gamma G_n(y,z) = (0)$

$\sum_{i=1}^{n} G_i(x,y)\alpha m\beta D_i(y,z) = 0$

By Lemma (2-3) we get

$\sum_{i=1}^{n} G_i(x,y)\alpha D_i(y,z) = 0$

Thus $G_n(x,y)\,\alpha D_n(y,z) = 0$

Lemma (2. 8):

Let M be a 2-torsion free semiprime Γ-ring aαyβz = aβyαz , two generalized symmetric higher bi-derivations D_n *and* G_n associated with two symmetric higher bi-derivations d_n *and* g_n respectively for n ∈N .Then D_n *and* G_n are orthogonal iff $D_n(x,y)\alpha g_n(y,z) = 0$ or $d_n(x,y)\alpha G_n(y,z) = 0$ for all $x,y,z\epsilon M, \alpha,\beta\epsilon\Gamma$ *and* $n\epsilon N$.

Proof:

Suppose that $D_n(x,y)\alpha g_n(y,z) = 0$

$D_n(x,y)\alpha g_n(y,z) = \sum_{i=1}^{n} D_i(x,y)\alpha g_i(y,z) = 0$ (1)

Replace z *by* $w\beta z$ in (1) for all w∈M we get

$\sum_{i=1}^{n} D_i(x,y)\alpha g_i(y,w\beta z) = 0$

$\sum_{i=1}^{n} D_i(x,y)\alpha g_i(y,w)\beta g_i(y,z) = 0$ (2)

Replace $g_i(y,z)$ by $G_i(y,z)$in (2) we get

$\sum_{i=1}^{n} D_i(x,y)\alpha g_i(y,w)\beta G_i(y,z) = 0$

By Lemma (2-3) we get

$\sum_{i=1}^{n} D_i(x,y)\alpha G_i(y,z) = 0$

$D_n(x,y)\alpha G_n(y,z) = 0$

By Lemma (2-7) we get D_n *and* G_n are orthogonal .

Similarly we if $d_n(x,y)\alpha G_n(y,z) = 0$ we get D_n *and* G_n are orthogonal .

Conversely , suppose that D_n *and* G_n are orthogonal.

By Lemma (2-7) we get
$D_n\ (x,y)\alpha G_n(y,z)=0$
$\sum_{i=1}^{n} D_i\ (x,y)\alpha G_i(y,z)=0$ (3)

Replace z by wβz in (3) for all wϵM we get
$\sum_{i=1}^{n} D_i(x,y)\alpha G_i(y,w\beta z)=0$
$\sum_{i=1}^{n} D_i\ (x,y)\alpha G_i(y,w)\beta g_i(y,z)=0$

By Lemma (2-3) we get
$\sum_{i=1}^{n} D_i(x,y)\alpha\ g_i(y,z)=0$
Hence $D_n\ (x,y)\alpha\ g_n(y,z)=0$

And replace x by wβx in (3) we get
$\sum_{i=1}^{n} D_i\ (\mathrm{w}\beta x,y)\alpha G_i(y,z)=0$
$\sum_{i=1}^{n} D_i(w,y)\beta d_i(x,y)\alpha G_i(y,z)=0$ (4)

Multiplication (4) by $d_i(x,y)\alpha G_i(y,z)\delta$ for all $\delta\epsilon\Gamma$ we get
$\sum_{i=1}^{n} d_i(x,y)\alpha G_i(y,z)\delta\ D_i(w,y)\beta d_i(x,y)\alpha G_i(y,z)=0$
Since M is $semiprime$ we get
$\sum_{i=1}^{n} d_i(x,y)\alpha G_i(y,z)=0$
Hence $d_n(x,y)\alpha G_n(y,z)=0$

Lemma (2. 9):

Let M be a 2-torsion free $semiprime$ Γ-ring aαyβz = aβyαz , two generalized symmetric higher bi-derivations D_n and G_n associated with two symmetric higher bi-derivations d_n and g_n respectively for all n ϵN . Then D_n and G_n are orthogonal if and only if $D_n(x,y)\alpha G_n(y,z)$=$d_n(x,y)\alpha G_n(y,z)$=0 for all x, y, zϵM , αϵΓ and n ϵN .

Proof:

Suppose that D_n and G_n are orthogonal
By Lemma (2-7) we get
$D_n(x,y)\alpha G_n(y,z)$=0 (1)

And by Lemma (2-8) we get
$d_n(x,y)\alpha G_n(y,z)=0$ (2)
From (1) and (2) we get $D_n(x,y)\alpha G_n(y,z)=d_n(x,y)\alpha G_n(y,z)=0$

Conversely , suppose that $D_n(x,y)\alpha G_n(y,z)=0$
By Theorem (2-7) we get
Hence D_n and G_n are orthogonal
Now, if $d_n(x,y)\alpha G_n(y,z)=0$
By Theorem (2-8) we get
D_n and G_n are orthogonal.

3. Main Results

In this section, we present and study some basic Theorems of orthogonal generalized symmetric higher bi-derivations on Γ-ring M.

Theorem (3. 1):

Let M is a 2-torsion free $semiprime$ Γ-ring aαyβz = aβyαz , D_n and G_n are orthogonal generalized symmetric higher bi-derivations associated with symmetric higher bi-derivations d_n and g_n respectively for all n ϵN. Then the following relations are hold for all x, y, z ϵM ,α, $\beta\epsilon\Gamma$ and nϵN.
(i) $\mathrm{D_n}(\mathrm{x},\mathrm{y})\alpha\mathrm{G_n}(\mathrm{y},\mathrm{z})=\mathrm{G_n}(\mathrm{x},\mathrm{y})\alpha\mathrm{D_n}(\mathrm{y},\mathrm{z})=0$ hence$\mathrm{D_n}\ (\mathrm{x},\mathrm{y})\alpha\ \mathrm{G_n}(\mathrm{y},\mathrm{z})+\ \mathrm{G_n}(\mathrm{x},\mathrm{y})\alpha\mathrm{D_n}(\mathrm{y},\mathrm{z})=0$.

(ii) d_n and G_n are orthogonal and $d_n(x,y)\alpha G_n(y,z)=G_n(x,y)\alpha d_n(y,z)=(0)$.
(iii) g_n and D_n are orthogonal and $g_n(x,y)\alpha D_n(y,z) = D_n(x,y)\alpha g_n(y,z) =(0)$.
(iv) d_n and g_n are orthogonal symmetric higher bi-derivations .
(v) $d_nG_n = G_nd_n = 0$ and $g_nD_n = D_ng_n = 0$.
(vi) $G_nD_n = D_nG_n = 0$.

Proof : (i)
Suppose that D_n *and* G_n are orthogonal
By Lemma (2-7) we get
$D_n(x,y)\alpha G_n(y,z) = 0$ *and* $G_n(x,y)\alpha D_n(y,z) = 0$
$D_n(x,y)\alpha G_n(y,z) = G_n(x,y)\alpha D_n(y,z) = 0$
Hence $D_n(x,y)\alpha G_n(y,z) + G_n(x,y)\alpha D_n(y,z) = 0$

Proof: (ii)
Suppose that D_n *and* G_n are orthogonal
By Lemma (2-8) we get
$d_n(x,y)\alpha G_n(y,z) = 0$ (1)
$\sum_{i=1}^{n} d_i(x,y)\alpha G_i(y,z) = 0$ (2)
Replace x *by* $x\beta w$ in (2) w ϵM ,β ϵΓ we get
$\sum_{i=1}^{n} d_i(x\beta w,y)\alpha G_i(y,z) = 0$
$\sum_{i=1}^{n} d_i(x,y)\beta d_i(w,y)\alpha G_i(y,z) = 0$ (3)
Replace $d_i(w,y)$ by m in (3) m ϵM we get
$\sum_{i=1}^{n} d_i(x,y)\beta m\alpha G_i(y,z) = 0$ (4)
And from (i) $G_n(x,y)\alpha D_n(y,z) = 0$
$\sum_{i=1}^{n} D_i(x,y)\alpha G_i(y,z) = 0$ (5)
Replace z *by* $w\beta z$ in (5) we get
$\sum_{i=1}^{n} G_i(x,y)\alpha D_i(y,w\beta z) = 0$
$\sum_{i=1}^{n} G_i(x,y)\alpha D_i(y,w)\beta d_i(y,z) = 0$
By Lemma (2-3) we get
$\sum_{i=1}^{n} G_i(x,y)\alpha d_i(y,z) = 0$
$G_n(x,y)\alpha\, d_n(y,z) = 0$ (6)
And by $\sum_{i=1}^{n} G_i(x,y)\alpha d_i(y,z) = 0$,replace z by w$\beta z$ we get
$\sum_{i=1}^{n} G_i(x,y)\alpha\, d_i(y,w\beta z) = 0$
$\sum_{i=1}^{n} G_i(x,y)\alpha d_i(y,w)\beta d_i(y,z) = 0$ (7)
Replace $\alpha d_i(y,w)\beta$ by $\beta d_i(w,y)\alpha$ in (7) we get
$\sum_{i=1}^{n} G_i(x,y)\beta d_i(w,y)\alpha d_i(y,z) = 0$ (8)
Replace $d_i(w,y)$ by m in (8) we get
$\sum_{i=1}^{n} G_i(x,y)\beta m\alpha d_i(y,z) = 0$ (9)
From (4) and (9) we get D_n *and* G_n are orthogonal
From (1)and (6) we get
$G_n(x,y)\alpha d_n(y,z) = d_n(x,y)\alpha G_n(y,z) = 0$

Proof: (iii)
Similarly way used in the proof of (ii)

Proof: (iv)
From (i) $D_n(x,y)\alpha G_n(y,z) = 0$
$\sum_{i=1}^{n} D_i(x,y)\alpha G_i(y,z) = 0$ (1)
Replacing x *by* $w\beta x$ *and* z *by* $w\gamma z$ *in*(1) *for all* $\gamma \in \Gamma$ we get
$\sum_{i=1}^{n} D_i(w\beta x,y)\alpha G_i(y,w\gamma z) = 0$
$\sum_{i=1}^{n} D_i(w,y)\beta d_i(x,y)\alpha G_i(y,w)\gamma g_i(y,z) = 0$ (2)
Replace $G_i(y,w)$ by m in (2) for all m ϵM we get
$\sum_{i=1}^{n} D_i(w,y)\beta d_i(x,y)\alpha m\gamma g_i(y,z) = 0$ (3)
Multiplication (3) by $d_i(x,y)\alpha m\gamma g_i(y,z)\delta$ for all $\delta\epsilon\Gamma$ we get

$\sum_{i=1}^{n} d_i(x,y)\alpha m\gamma g_i(y,z)\delta D_i(w,y)\beta d_i(x,y)\alpha m\gamma g_i(y,z) = 0$
Since M is $semiprime$ we get
$\sum_{i=1}^{n} d_i(x,y)\alpha m\gamma g_i(y,z) = 0$
$d_n(x,y)\Gamma M\Gamma g_n(y,z) = (0)$
Hence d_n and g_n are orthogonal symmetric higher bi-derivations.

Proof: (v)
Since by (ii) $d_n(x,y)\alpha G_n(y,z) = 0$
$G_n(d_n(x,y)\alpha G_n(y,z),m) = 0$ for all $m\epsilon M$
$\sum_{i=1}^{n} G_i(d_i(x,y)\alpha G_i(y,z),m) = 0$ (1)
Replace x by $x\beta w$ in (1) for all w ϵM , β ϵΓ we get
$\sum_{i=1}^{n} G_i(d_i(x\beta w\,,y)\alpha G_i(y,z),m) = 0$
$\sum_{i=1}^{n} G_i(d_i(x\,,y)\beta d_i(w,y)\alpha G_i(y,z),m) = 0$
$\sum_{i=1}^{n} G_i(d_i(x,y),m)\beta g_i(d_i(w,y),m)\alpha g_i(G_i(y,z),m) = 0$ (2)
Replace $g_i(G_i(y,z),m)$ by $G_i(d_i(x,y),m)$ in (2) we get
$\sum_{i=1}^{n} G_i(d_i(x,y),m)\beta g_i(d_i(w,y),m)\alpha G_i(d_i(x,y),m) = 0$
Since M is $semiprime$ we get
$\sum_{i=1}^{n} G_i(d_i(x,y),m) = 0$
Thus $G_n d_n = 0$ (3)
And by (ii) $G_n(x,y)\alpha d_n(y,z) = 0$
$d_n(G_n(x,y)\alpha d_n(y,z),m) = 0$
$\sum_{i=1}^{n} d_i(G_i(x,y)\alpha d_i(y,z),m) = 0$ (4)
Replace xby $x\delta w$ in (4) for all $\delta\epsilon\Gamma$ we get
$\sum_{i=1}^{n} d_i(G_i(x\delta w,y)\alpha d_i(y,z),m) = 0$
$\sum_{i=1}^{n} d_i(G_i(x,y)\delta g_i(w,y)\alpha d_i(y,z),m) = 0$
$\sum_{i=1}^{n} d_i(G_i(x,y),m)\delta d_i(g_i(w,y),m)\alpha d_i(d_i(y,z),m) = 0$ (5)
Replace $d_i(d_i(y,z),m)$ by $d_i(G_i(x,y),m)$ in (5) we get
$\sum_{i=1}^{n} d_i(G_i(x,y),m)\delta d_i(g_i(w,y),m)\alpha d_i(G_i(x,y),m) = 0$
Since M is $semiprime$ we get
$\sum_{i=1}^{n} d_i(G_i(x,y),m) = 0$
Thus $d_n G_n = 0$ (6)
From (3) and (6) we get
$G_n d_n = d_n G_n = 0$
Similarly way to prove that $D_n g_n = g_n D_n = 0$.

Proof: (vi)
Since D_n and G_n are orthogonal
$D_n(x,y)\Gamma M\Gamma G_n(y,z) = (0)$
$G_n(D_n(x,y)\Gamma M\Gamma G_n(y,z),r) = (0)$ for all rϵM
$\sum_{i=1}^{n} G_i(D_i(x,y)\alpha m\beta G_i(y,z),r) = 0$
$\sum_{i=1}^{n} G_i(D_i(x,y),r)\alpha g_i(m,r)\beta g_i(G_i(y,z),r) = 0$ (1)
Replace $g_i(G_i(y,z),r)$ by $G_i(D_i(x,y),r)$ we get
$\sum_{i=1}^{n} G_i(D_i(x,y),r)\alpha g_i(m,r)\beta G_i(D_i(x,y),r) = 0$
Since M is $semiprime$ we get
$\sum_{i=1}^{n} G_i(D_i(x,y),r) = 0$
Thus $G_n D_n = 0$ ….(2)
And by $G_n(x,y)\Gamma M\Gamma D_n(y,z) = (0)$
$D_n(G_n(x,y)\Gamma M\Gamma D_n(y,z),r) = (0)$
$\sum_{i=1}^{n} D_i(G_i(x,y)\alpha m\beta D_i(y,z),r) = 0$
$\sum_{i=1}^{n} D_i(G_i(x,y),r)\alpha d_i(m,r)\beta d_i(D_i(y,z),r) = 0$ (3)
Replace $d_i(D_i(y,z),r)$ by $D_i(G_i(x,y),r)$ in (3) we get
$\sum_{i=1}^{n} D_i(G_i(x,y),r)\alpha d_i(m,r)\beta D_i(G_i(x,y),r) = 0$
Since M is $semiprime$ we get

$\sum_{i=1}^{n} D_i(G_i(x,y),r) = 0$
Thus $D_nG_n = 0$ (4)

From (2) and (4) we get
$G_nD_n = D_nG_n = 0$

Theorem (3.2):
Let M be 2-torsion freesemiprime Γ-ring aαyβz = aβyαz, D_n and G_n are generalized symmetric higher bi-derivations associated with symmetric higher bi-derivations d_n and g_n respectively for all nϵN. Then the following relations are equivalent for all x, yϵM and α, βϵΓ:

$(i) D_n$ and G_n are orthogonal
$(ii) D_n(x,y)\alpha G_n(y,z) + G_n(x,y)\alpha D_n(y,z) = 0$
$(iii) d_n(x,y)\alpha G_n(y,z) + g_n(x,y)\alpha D_n(y,z) = 0$

Proof: $(i) \Leftrightarrow (ii)$
Suppose that D_n and G_n are orthogonal
By Theorem (3-1) (i) we get
$D_n(x,y)\alpha G_n(y,z) = G_n(x,y)\alpha D_n(y,z) = 0$
Hence $D_n(x,y)\alpha G_n(y,z) + G_n(x,y)\alpha D_n(y,z) = 0$
$Conversly, Let$ $D_n(x,y)\alpha G_n(y,z) + G_n(x,y)\alpha D_n(y,z) = 0$
By Lemma (2-6) we get
HenceD_n and G_n are orthogonal
$(i) \Leftrightarrow (iii)$
Suppose that D_n and G_n are orthogonal
By Lemma (2-8) we get
$d_n(x,y)\alpha G_n(y,z) = 0$ (1)
And by Theorem (3-1) (i) we get
$G_n(x,y)\alpha D_n(y,z) = 0$
$\sum_{i=1}^{n} G_i(x,y)\alpha D_i(y,z) = 0$ (2)
Replace x by $t\beta x$ in (2) for t ϵM we get
$\sum_{i=1}^{n} G_i(t\beta x,y)\alpha D_i(y,z) = 0$
$\sum_{i=1}^{n} G_i(t,y)\beta g_i(x,y)\alpha D_i(y,z) = 0$ (3)
Multiplication (3) by $g_i(x,y)\alpha D_i(y,z)\delta$ for all δ $\epsilon\Gamma$ we get
$\sum_{i=1}^{n} g_i(x,y)\alpha D_i(y,z)\delta G_i(t,y)\beta g_i(x,y)\alpha D_i(y,z) = 0$

Since M $semiprime$ we get
$\sum_{i=1}^{n} g_i(x,y)\alpha D_i(y,z) = 0$
$g_n(x,y)\alpha D_n(y,z) = 0$ (4)
From (1) and (4) we get
$d_n(x,y)\alpha G_n(y,z) + g_n(x,y)\alpha D_n(y,z) = 0$
$Conversely$ $, Let$ $d_n(x,y)\alpha G_n(y,z) + g_n(x,y)\alpha D_n(y,z) = 0$
$\sum_{i=1}^{n} d_i(x,y)\alpha G_i(y,z) + g_i(x,y)\alpha D_i(y,z) = 0$ (5)
Replace x by $x\gamma t$ in (5) for all γ $\epsilon\Gamma$ we get
$\sum_{i=1}^{n} d_i(x\gamma t,y)\alpha G_i(y,z) + g_i(x\gamma t,y)\alpha D_i(y,z) = 0$
$\sum_{i=1}^{n} d_i(x,y)\gamma d_i(t,y)\alpha G_i(y,z) + g_i(x,y)\gamma g_i(t,y)\alpha D_i(y,z) = 0$ (6)
Replacing $d_i(x,y)$ by $D_i(x,y)and$ $g_i(x,y)$ by $G_i(x,y)$in (6) we get
$\sum_{i=1}^{n} D_i(x,y)\gamma d_i(t,y)\alpha G_i(y,z) + G_i(x,y)\gamma g_i(t,y)\alpha D_i(y,z) = 0$ (7)
Replace$d_i(t,y)$ by $g_i(t,y)$ in (7) we get
$\sum_{i=1}^{n} D_i(x,y)\gamma g_i(t,y)\alpha G_i(y,z) + G_i(x,y)\gamma g_i(t,y)\alpha D_i(y,z) = 0$
By Lemma (2-4) we get
$\sum_{i=1}^{n} D_i(x,y)\gamma g_i(t,y)\alpha G_i(y,z)$=$\sum_{i=1}^{n} G_i(x,y)\gamma g_i(t,y)\alpha D_i(y,z)$= 0
Hence D_n and G_n are orthogonal

Theorem (3. 3):

Let M be 2-torsion free *semiprime* Γ-ring aαyβz = aβyαz , D_n and G_n are generalized symmetric higher bi-derivations associated with symmetric higher bi-derivations d_n *and* g_n respectively for all nϵ N. Then D_n and G_n are orthogonal iff $D_n(x,y)\alpha G_n(y,z)$ =0 for all $x,y,z\epsilon M$, $\alpha,\beta\epsilon\Gamma$ and $d_n G_n = d_n g_n$=0.

Proof:

Suppose that D_n and G_n are orthogonal
By Theorem (2-7) we get
$D_n(x,y)\alpha G_n(y,z) = 0$ (1)
And by Theorem (3-1) (i) we get
$G_n(x,y)\alpha d_n(y,z) = 0$
$d_n(\, G_n(x,y)\alpha d_n(y,z), m) = 0$
$\sum_{i=1}^{n} d_i(\, G_n(x,y)\alpha d_i(y,z), m) = 0$ (2)
Replace $x\ by\ x\beta t$ in (2) for all $t\epsilon M$ we get
$\sum_{i=1}^{n} d_i(G_i(x\beta t,y)\alpha d_i(y,z), m) = 0$
$\sum_{i=1}^{n} d_i(G_i(x,y)\beta g_i(t,y)\alpha d_i(y,z), m) = 0$
$\sum_{i=1}^{n} d_i(G_i(x,y),m)\beta d_i(g_i(t,y),m)\alpha d_i(d_i(y,z),m) = 0$ (3)
Replace $d_i(d_i(y,z),m)$ by $d_i(G_i(x,y),m)$ in (3) we get
$\sum_{i=1}^{n} d_i(G_i(x,y),m)\beta d_i(g_i(t,y),m)\alpha d_i(G_i(x,y),m) = 0$
Since M is *semiprime* we get
$\sum_{i=1}^{n} d_i(G_i(x,y),m) = 0$
$d_n G_n = 0$ (4)
Also by Theorem (3-1) (iv) we get
$g_n(x,y)\Gamma M\Gamma d_n(y,z) = (0)$
$d_n(g_n(x,y)\Gamma M\Gamma d_n(y,z), r) = (0)$ for all $r\epsilon M$
$\sum_{i=1}^{n} d_i(g_i(x,y)\alpha m\beta d_i(y,z), r) = 0$
$\sum_{i=1}^{n} d_i(g_i(x,y),r)\alpha d_i(m,r)\beta d_i(d_i(y,z),r) = 0$ (5)
Replace $d_i(y,z)$ by $g_i(x,y)$ in (5) we get
$\sum_{i=1}^{n} d_i(g_i(x,y),r)\alpha d_i(m,r)\beta d_i(g_i(x,y),r) = 0$
Since M is *semiprime* we get
$\sum_{i=1}^{n} d_i(g_i(x,y),r) = 0$
$d_n g_n = 0$ (6)
From (1) and (4), (6) we get
$D_n(x,y)\alpha G_n(y,z) = 0\ and\ d_n G_n = d_n g_n = 0$
Conversely , suppose that $D_n(x,y)\alpha G_n(y,z) = 0$ (7)
And $d_n G_n = 0$
$(d_n G_n)(x\alpha y,z) = 0$
$\sum_{i=1}^{n} d_i(G_i(x\alpha y,z),m) = 0$ for all m ϵM
$\sum_{i=1}^{n} d_i(G_i(x,z)\alpha g_i(y,z),m) = 0$
$\sum_{i=1}^{n} d_i(G_i(x,z),m)\alpha d_i(g_i(y,z),m) = 0$ (8)
Replacing $(G_i(x,z),m)$ by (x, y) and $d_i(g_i(y,z),m)$ by $G_i(y,z)$ in (8) we get
$\sum_{i=1}^{n} d_i(x,y)\alpha G_i(y,z) = 0$
$d_n(x,y)\alpha G_n(y,z) = 0$ (9)
From (7) and (9) we get
$D_n(x,y)\alpha G_n(y,z) = d_n(x,y)\alpha G_n(y,z) = 0$
By Lemma (2-9) we get $D_n\ and\ G_n$ are orthogonal

Theorem (3. 4):

Let M be a 2-torsion free *semiprime* Γ-ring such that aαyβz = aβyαz and D_n be a generalized symmetric higher bi-derivations associated with symmetric higher bi-derivations d_n for all nϵN. If $D_n(x,y)\alpha D_n(y,z) = 0$ for all x, y, z ϵM , $\alpha,\beta\epsilon\Gamma$,n ϵN then $D_n = d_n = 0$.

Proof:

Suppose that $D_n(x,y)\alpha D_n(y,z)=0$

$\sum_{i=1}^{n} D_i(x,y)\alpha D_i(y,z)=0$ (1)

Replace z *by* $t\beta z$ *in* (1) for all $t\,\epsilon M$, $\beta\,\epsilon\Gamma$ we get

$\sum_{i=1}^{n} D_i(x,y)\alpha D_i(y,t\beta z)=0$

$\sum_{i=1}^{n} D_i(x,y)\alpha D_i(y,t)\beta d_i(y,z)=0$

By Lemma (2-3) we get

$\sum_{i=1}^{n} D_i(x,y)\alpha d_i(y,z)=0$ (2)

Multiplication (2) by $di(y,z)\delta$ for all $\delta\,\epsilon\Gamma$ we get

$\sum_{i=1}^{n} d_i(y,z)\delta\, D_i(x,y)\alpha d_i(y,z)=0$

Since M is *semiprime* we get

$\sum_{i=1}^{n} d_i(y,z)=0$

$d_n=0$ (3)

And multiplication (1) by δ D_i(x, y) we get

$\sum_{i=1}^{n} D_i(x,y)\alpha D_i(y,z)\delta D_i(x,y)=0$

Since M is *semiprime* we get

$\sum_{i=1}^{n} D_i(x,y)=0$

D_n=0 (4)

From (3) and (4) we get

$D_n=d_n=0$

Theorem (3.5):

Let M be a 2-torsion free *semiprime* Γ-ring. Let U be an ideal of M and V= Ann. (U) .If $(D_n\,,d_n)$is generalized symmetric higher bi-derivations for all nϵN such that D_n(M) , d_n(M) CU then D_n(V)=d_n (V) = 0 .

Proof:

If x,yϵV , $\alpha\epsilon\Gamma$ then (x y) α U =0

By hypothesis we have

d_n (M) ⊂U ⟹ d_n (U) ⊂ U

Hence $0=D_n\,(x\alpha z,y)$

$0=\sum_{i=1}^{n} D_i\,(x\alpha z,y)$

$0=\sum_{i=1}^{n} D_i\,(x,y)\alpha d_i\,(z,y)$ (1)

Multiplication (1) by $\beta D_i\,(x,y)$ for all $\beta\epsilon\Gamma$ we get

$0=\sum_{i=1}^{n}\, D_i\,(x,y)\alpha d_i(z,y)\,\beta D_i\,(x,y)$

Since M is *semiprime* we get

$0=\sum_{i=1}^{n}\, D_i\,(x,y)\,\epsilon\, U\cap V$

$D_n\,(x,y)=0$

Similarly, since (x y)α U =0 for all x,yϵV , $\alpha\epsilon\Gamma$

$0=d_n(x\alpha z,y)$

$0=\sum_{i=1}^{n} d_i(x\alpha z,y)$

$0=\sum_{i=1}^{n} d_i(x,y)\alpha d_i(z,y)$ (2)

Multiplication (2) by $\beta d_i(x,y)$ we get

$0=\sum_{i=1}^{n} d_i(x,y)\alpha d_i(y,z)\beta d_i(x,y)$

Since M is *semiprime* we get

$0=\sum_{i=1}^{n} d_i(x,y)\;\epsilon\; U\cap V$

$d_n(x,y)=0$

Theorem (3.6):

Let M be 2-torsion free semiprime Γ-ring $x\alpha y\beta z=x\beta y\alpha z$ for all $x,y,z\epsilon M$ and $\alpha,\beta\epsilon\Gamma$, D_n and G_n are generalized symmetric higher bi-derivations associated with symmetric higher bi-derivations d_n and g_n respectively for all n ϵ N .Then D_n and g_n as well as G_n and d_n are orthogonal iff $D_n=d_n=0$ or $G_n=g_n$= 0.

Proof:

Suppose that D_n *and* g_n as well as G_n *and* d_n are orthogonal

By Theorem (3-1) (iii) we get

$D_n(x,y)\alpha g_n(y,z) = 0$

$\sum_{i=1}^{n} D_i(x,y)\alpha g_i(y,z) = 0$ (1)

Multiplication (1) by $\beta D_i(x,y)$ for all $\beta \epsilon\Gamma$ we get

$\sum_{i=1}^{n} D_i(x,y)\alpha g_i(y,z)\beta D_i(x,y) = 0$

Since M is *semiprime* we get

$\sum_{i=1}^{n} D_i(x,y) = 0$

$D_n = 0$ (2)

And by Theorem (3-1) (ii) we get

$d_n(x,y)\alpha G_n(y,z) = 0$

$\sum_{i=1}^{n} d_i(x,y)\alpha G_i(y,z) = 0$ (3)

Multiplication (3) by $\beta d_i(x,y)$ we get

$\sum_{i=1}^{n} d_i(x,y)\alpha G_i(y,z)\, \beta d_i(x,y) = 0$

Since M is *semiprime* we get

$\sum_{i=1}^{n} d_i(x,y) = 0$

$d_n = 0$ (4)

Now, by Theorem (3-1) (iii) we get

$g_n(x,y)\alpha D_n(y,z) = 0$

$\sum_{i=1}^{n} g_i(x,y)\alpha D_i(y,z) = 0$ (5)

Multiplication (5) by $\beta g_i(x,y)$ we get

$\sum_{i=1}^{n} g_i(x,y)\alpha D_i(y,z)\beta g_i(x,y) = 0$

Since M is *semiprime* we get

$\sum_{i=1}^{n} g_i(x,y) = 0$

$g_n = 0$ (6)

And by Theorem (3-1) (ii) we get

$G_n(x,y)\alpha d_n(y,z) = 0$

$\sum_{i=1}^{n} G_i(x,y)\alpha d_i(y,z) = 0$ (7)

Multiplication (7) by $\beta G_i(x,y)$ we get

$\sum_{i=1}^{n} G_i(x,y)\alpha d_i(y,z)\, \beta G_i(x,y) = 0$

Since M is *semiprime* we get

$\sum_{i=1}^{n} G_i(x,y) = 0$

$G_n = 0$ (8)

From (2) , (4) and (6) ,(8) we get

$D_n = d_n = 0 \text{ or } G_n = g_n = 0$

$Conversly,\ suppose\ that\ D_n = d_n = 0 \text{ or } G_n = g_n = 0$

$D_n(x\alpha z, y) = 0$

$g_n(D_n(x\alpha z, y), m) = 0$

$\sum_{i=1}^{n} g_i(D_i(x\alpha z, y), m) = 0$

$\sum_{i=1}^{n} g_i(D_i(x,y)\alpha d_i(z,y), m) = 0$

$\sum_{i=1}^{n} g_i(D_i(x,y), m)\alpha g_i(d_i(z,y), m) = 0$ (9)

Replacing $g_i(D_i(x,y),m)$ by (x,y) and $(d_i(z,y),m)$ by (y,z) in (9) we get

$\sum_{i=1}^{n} D_i(x,y)\alpha g_i(y,z) = 0$

$D_n(x,y)\alpha g_n(y,z) = 0$

By Theorem (3-1) (iii)

Hnce D_n *and* g_n are orthogonal

Similarly , if $G_n = g_n = 0$ we get

Hence G_n *and* d_n are orthogonal

Theorem (3.7):

Let M be a 2-torsion free semiprime Γ-ring, D_n and G_n are generalized symmetric higher bi-derivations for all nϵN.Suppose that $D_n\Gamma G_n$=$G_n\Gamma D_n$, then $D_n - G_n$ and $D_n + G_n$ are orthogonal .

Proof:

Suppose that $D_n\Gamma G_n = G_n\Gamma D_n$, then for $x, y \in M$:

$= [(D_n + G_n)\Gamma(D_n - G_n) + (D_n - G_n)\Gamma(D_n + G_n)](x, y)$

$= [(D_n + G_n)\Gamma(D_n - G_n)](x, y) + [(D_n - G_n)\Gamma(D_n + G_n)](x, y)$

$= \sum_{i=1}^{n}[(D_i + G_i)\alpha(D_i - G_i)](x, y) + \sum_{i=1}^{n}[(D_i - G_i)\alpha(D_i + G_i)] (x, y)$ for all $\alpha\epsilon\Gamma$

$= \sum_{i=1}^{n}(D_i\alpha D_i - D_i\alpha G_i + G_i\alpha D_i - G_i\alpha G_i)(x, y) + \sum_{i=1}^{n}(D_i\alpha D_i + D_i\alpha G_i - G_i\alpha D_i - G_i\alpha G_i) (x, y) =$
$\sum_{i=1}^{n} D_i(x, y)\alpha D_i(x, y) - D_i(x, y)\alpha G_i(x, y) + G_i(x, y)\alpha D_i(x, y) - G_i(x, y)\alpha G_i(x, y) +$
$\sum_{i=1}^{n} D_i(x, y)\alpha D_i(x, y) + D_i(x, y)\alpha G_i(x, y) - G_i(x, y)\alpha D_i(x, y) - G_i(x, y)\alpha G_i(x, y)$

Therefore $\sum_{i=1}^{n}[(D_i + G_i)\alpha(D_i - G_i)](x, y) + \sum_{i=1}^{n}[(D_i - G_i)\alpha(D_i + G_i)] = 0$

By Lemma (2-4) we get

$\sum_{i=1}^{n}[(D_i + G_i)\alpha(D_i - G_i)](x, y) = 0 = \sum_{i=1}^{n}[(D_i - G_i)\alpha(D_i + G_i)](x, y)$

Thus $D_n - G_n$ and $D_n + G_n$ are orthogonal

References

1. Nobusawa, N. **1964.** On Generalization of the Ring Theory. *Osaka J. Math*., **1**: 81-89.
2. Barnes, W.E. **1966.** On The Γ-Rings of Nobusawa. *Pacific J. Math*. **18**: 411-422.
3. Kyuno, S. **1978.** On Prime Gamma Rings. *Pacific J. of Math*., **75**: 185-190.
4. Jing, F.J. **1987.** On Derivations of Γ-Ring. *QUFU Shi Fan Daxue Xuebeo Ziran Kexue Ban*, **13**(4): 159-161.
5. Sapanci, M. and Nakajima, A. **1997.** Jordan Derivations on Complately Prime Gamma Rings. *Math., Japoncia* , **46**(1): 47-51.
6. Cenven, Y. and M. A. Ozturk, M. A. **2004.** On Jordan Generalized Derivations in Gamma Rings. *Hacettepe J. of Mathematics and Statistics*, **33**: 11-14.
7. Ashraf, M. and Jamal, M.R. 2010. Orthogonal Derivations in Gamma Ring. Advance in Algebra, **3**(1): 1-6.
8. Ozturk, M.A., Sapanci, M., Soyturk, M. and Kim, K.H. **2000.** Symmetric Bi-Derivation Prime Gamma Rings. *Scientiae Mathematicae* , **3**(2): 273-281.
9. Marir, A.M. and Salih, S.M. **2016.** Higher Bi-Derivations on Prime Gamma Rings. M.SC. Thesis Education College, AL-Mustansiriya Univ.
10. Chakraborty, S. and Paul, A.C. **2008**. On Jordan K-derivations of 2 –Torsion free Prime Γ N-Rings. *Punjap University J. of Math*., **40**: 97-101.
11. Dutta, T.K. and Sardar, S.K. **2000.** Semiprime Ideal and Irreducible Ideal of Γ-Semi rings. *Novi Sad J. Math*. **30**(1): 97-108.