



The dependence of spectral parameters in the cross-sectional equation of coherent and incoherent radiation on energy using a mathematical method

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Abstract

The dependence of the cross-section of the coherent and incoherent radiation peaks in the X-ray absorption experiment of different energies (20-800 Kev) was investigated. Cross-sectional dependence on the atomic number Z was included from the published data for (8) elements, ranging from carbon to silver (C-Ag). The proportional constant K was obtained between (σ_c/σ_i), with the atomic number Z from (6-47). The results show that the value of K exponentially changes with energy.

Keywords: energy, mathematics, cross-sectional, X-ray, elements.

اعتماد المعاملات الطيفية في معادلة المقطع العرضي للأشعة المشاكهة وغير المشاكهة على الطاقة بطرق رياضية

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الخلاصة

اعتماد المقطع العرضي لقمة الأشعة المشاكهة وغير المشاكهة في تجربة امتصاص الأشعة السينية لمختلف الطاقات من (800-20) KeV . في هذا البحث تضمن اعتماد المقطع العرضي على العدد الذري Z من البيانات المنشورة لثمانية عناصر تتراوح من الكاربون الى الفضة (C-Ag) ، حيث تم الحصول على ثابت التناسب K من إجمالي المقطع العرضي الذري المشاكهة وغير المشاكهة (σ_c/σ_i) مع العدد الذري Z من (6-42) مبيناً بان قيمة K تتغير اسيا مع الطاقة.

Introduction

Information on the constituents of trace elements in samples of biological, industrial, environmental, and geological origin is important for a variety of reasons. A number of analytical techniques are being used for this purpose. As compared to electron or charged particle excitation sources, X-ray excitation does not cause loss of volatile elements or other chemical changes. In X-ray fluorescence (XRF) analysis, the precise determination of trace element content in a specimen requires, among other factors, a correction factor that takes account of the absorption of primary and fluorescence X-ray in the specimen [1, 2]. Coherent and incoherent radiation scattering is a type of

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interaction of γ -rays and x-rays with matter. Coherent and Compton scattering are the basic processes for obtaining information about the structural properties of a substance. The coupled electrons of the atom, which dominate coherent scattering for most x-ray and low-energy γ modes, contribute to coherent scattering. In the physics of condensed matter, coherent and Compton scattering have also played a central role in understanding the excited states of many important systems [3, 4]. Coherent scattering of photons by atoms includes Rayleigh scattering, Thomson nuclear scattering, Delbrook scattering, and nuclear resonance scattering. Non-coherent scattering is one of the main processes by which γ -rays interact with a substance in the energy range from 0.1 to 5 MeV [2, 3, 14]. Coherent and incoherent scattering sections are used in such diverse fields as medical X-ray technology, power reactor protection, industrial radiation treatment, and analysis of nuclear physics experiments (4). Van Espen *et al.* [5] used coherent to incoherent radiation scattering ratio to determine the effective sample weight. Van Dyck and Van Grieken [6 – 9] used absorption correction via scattered radiation in the energy dispersive X-ray fluorescence (EDXRF) analysis of samples of varying composition. Tursucu *et al.* [11] determined the effective atomic number of rare-earth samples by using the scattering intensity ratio method. This ratio is used to determine the mean atomic number Z of the sample and expressed as the ratio of total coherent and incoherent atomic cross-section (σ_c/σ_i), with the following simple formula [10, 12, 13]:

$$\sigma_c/\sigma_i = k Z^{(n-1)} \quad (1)$$

where K is an energy dependent constant according to Tomic *et al* [12], who used an M_o -tube coupled to an M_o foil and 90° detection angle, where (n-1) was found to be nearly 1.65. Since excitation energies are used nowadays in energy dispersed X-ray spectrometry, it is believed as desirable here to have a graph in which the variation of K and (n) parameters is displayed with the exciting radiation. It seems also desirable here to find out the explicitly that the cross- section of the coherent and non-coherent radiation peaks in the X-ray absorption experiment varies with Z for photon energies ranging from (20-800 Kev).

Theoretical Calculation

Values of the atomic cross-sections σ_c and σ_i are shown in Tables 1-13, taken from Hubbell *et al.* [11, 13] for the energies in the range of (20-800kev) which are supposed to strike the sample elements with an atomic number in the range of (6-47). Taking the natural logarithm, equation (1) may be written as:

$$\ln(\sigma_c/\sigma_i) = \ln K + (n-1) \ln Z \quad (2)$$

Table 1-Values of the atomic cross-sections σ_c and σ_i for an energy value of E=20kev [11, 13].

Z	ln Z	σ_c	σ_i	$\frac{\sigma_c}{\sigma_i}$	$\frac{\sigma_c}{\sigma_i} \ln \frac{\sigma_c}{\sigma_i}$
6	1.7917	1.29	3.19	0.4044	-0.9054
13	2.5649	9.18	6.15	1.4927	0.4006
20	2.9957	25.5	8.82	2.8912	1.0617
26	3.258	48	10.8	4.4444	1.4917
29	3.367	64	11.6	5.5172	1.7079
35	3.555	102	13.2	7.7273	2.0448
42	3.737	158	15.2	10.3947	2.3413
47	3.85	209	16.9	12.5904	2.5329

Table 2-Values of the atomic cross-sections σ_c and σ_i for an energy value of E=40kev [11, 13].

Z	$\ln Z$	σ_c	σ_i	$\frac{\sigma_c}{\sigma_i}$	$\frac{\sigma_c}{\ln \sigma_i}$
6	1.7917	0.409	3.3	0.1239	-2.088
13	2.5649	3.08	6.7	0.4597	-0.7772
20	2.9957	8.66	9.87	0.8774	-0.1308
26	3.258	16.7	12.4	1.3468	0.2977
29	3.367	22.4	13.6	1.6471	0.499
35	3.555	36.7	15.9	2.3082	0.8365
42	3.737	57.8	18.5	3.1243	1.1392
47	3.85	76.3	20.3	3.7586	1.3241

Table 3-Values of the atomic cross-sections σ_c and σ_i for an energy value of E=60kev [11, 13].

Z	$\ln Z$	σ_c	σ_i	$\frac{\sigma_c}{\sigma_i}$	$\frac{\sigma_c}{\ln \sigma_i}$
6	1.7917	0.196	3.19	0.06144	-2.7897
13	2.5649	1.52	6.65	0.2286	-1.4759
20	2.9957	4.42	9.91	0.4460	-0.8047
26	3.258	8.51	12.6	0.6754	0.3925
29	3.367	11.4	13.9	0.8201	-0.1983
35	3.555	18.6	16.4	1.1341	0.1259
42	3.737	29.6	19.3	1.5337	0.4277
47	3.85	39.6	21.2	1.8679	0.6248

Table 4-Values of the atomic cross-sections σ_c and σ_i for an energy value of E=80kev [11, 13]

Z	$\ln Z$	σ_c	σ_i	$\frac{\sigma_c}{\sigma_i}$	$\frac{\sigma_c}{\ln \sigma_i}$
6	1.7917	0.114	3.05	0.03738	-32867
13	2.5649	0.898	6.45	0.1392	-1.9717
20	2.9957	2.64	9.69	0.2724	-1.3003
26	3.258	5.13	12.4	0.4137	-0.8826
29	3.367	6.86	13.7	0.5007	-0.6917
35	3.555	11.2	16.2	0.6914	-0.3691
42	3.737	17.8	19.2	0.9271	-0.0757
47	3.85	23.9	21.2	1.1274	0.1199

Table 5-Values of the atomic cross-sections σ_c and σ_i for an energy value of E=100kev [11, 13].

Z	ln Z	σ_c	σ_i	$\frac{\sigma_c}{\sigma_i}$	$\frac{\sigma_c}{\ln \sigma_i}$
6	1.7917	0.0745	2.92	0.02551	-3.6685
13	2.5649	0.591	6.22	0.09502	-2.3537
20	2.9957	1.74	9.39	0.1853	-1.6858
26	3.258	3.37	12.0	0.2808	-1.2700
29	3.367	4.52	13.3	0.3398	-1.0793
35	3.555	7.39	15.9	0.4648	-0.7662
42	3.737	11.8	18.8	0.6277	-0.4658
47	3.85	15.8	20.9	0.7560	-0.2797

Table 6-Values of the atomic cross-sections σ_c and σ_i for an energy value of E=150kev [11, 13].

Z	ln Z	σ_c	σ_i	$\frac{\sigma_c}{\sigma_i}$	$\frac{\sigma_c}{\ln \sigma_i}$
6	1.7917	0.0339	2.65	0.01279	-4.3589
13	2.5649	0.276	5.68	0.04349	-3.1353
20	2.9957	0.810	8.64	0.09375	-2.3671
26	3.258	1.57	11.1	0.1414	-1.9559
29	3.367	2.10	12.4	0.1694	-1.7758
35	3.555	3.45	14.8	0.2331	-1.4563
42	3.737	5.53	17.7	0.3124	-1.1634
47	3.85	7.42	19.7	0.3766	-0.9764

Table 7-Values of the atomic cross-sections σ_c and σ_i for an energy value of E=200kev [11, 13].

Z	ln Z	σ_c	σ_i	$\frac{\sigma_c}{\sigma_i}$	$\frac{\sigma_c}{\ln \sigma_i}$
6	1.7917	0.0193	2.43	0.007942	-4.8355
13	2.5649	0.157	5.23	0.03002	-3.5059
20	2.9957	0.467	7.99	0.05845	-2.8396
26	3.258	0.907	10.3	0.08806	-2.4298
29	3.367	1.21	11.5	0.1052	-2.2517
35	3.555	1.99	13.8	0.1442	-1.9365
42	3.737	3.19	16.5	0.1933	-1.6433
47	3.85	4.28	18.4	0.2326	-0.4584

Table 8 -Values of the atomic cross-sections σ_c and σ_i for an energy value of E=300kev [11, 13].

Z	$\ln Z$	σ_c	σ_i	$\frac{\sigma_c}{\sigma_i}$	$\frac{\sigma_c}{\ln \sigma_i}$
6	1.7917	0.00853	2.12	0.004024	-5.5156
13	2.5649	0.701	4.57	0.1534	-4.1773
20	2.9957	0.209	7.0	0.02986	-3.05113
26	3.258	0.407	9.08	0.04482	-3.1050
29	3.367	0.544	10.1	0.05386	-2.9213
35	3.555	0.892	12.2	0.07311	-2.6157
42	3.737	1.43	14.6	0.09795	-2.3233
47	3.85	1.93	16.3	0.1184	-2.1336

Table 9-Values of the atomic cross-sections σ_c and σ_i for an energy value of E=400kev [11, 13].

Z	$\ln Z$	σ_c	σ_i	$\frac{\sigma_c}{\sigma_i}$	$\frac{\sigma_c}{\ln \sigma_i}$
6	1.7917	0.00481	1.9	0.002532	-5.9789
13	2.5649	0.0396	4.1	0.009659	-4.6399
20	2.9957	0.118	6.3	0.01873	-3.9776
26	3.258	0.230	8.17	0.02815	-3.5701
29	3.367	0.308	9.10	0.03385	-3.3859
35	3.555	0.505	11.0	0.04591	-3.0811
42	3.737	0.812	13.1	0.06198	-2.7809
47	3.85	1.09	14.7	0.07415	-2.6017

Table 10- Values of the atomic cross-sections σ_c and σ_i for an energy value of E=500kev [11, 13].

Z	$\ln Z$	σ_c	σ_i	$\frac{\sigma_c}{\sigma_i}$	$\frac{\sigma_c}{\ln \sigma_i}$
6	1.7917	0.00308	1.73	0.0017803	-6.3309
13	2.5649	0.0255	3.75	0.0068	-4.9908
20	2.9957	0.0761	5.76	0.01321	-4.3266
26	3.258	0.148	7.47	0.01981	-3.9214
29	3.367	0.198	8.33	0.02377	-3.7394
35	3.555	0.324	10.0	0.0324	-3.4296
42	3.737	0.522	12.0	0.0435	-3.135
47	3.85	0.702	13.5	0.052	-2.9565

Table 11 -Values of the atomic cross-sections σ_c and σ_i for an energy level of E=600kev [11, 13].

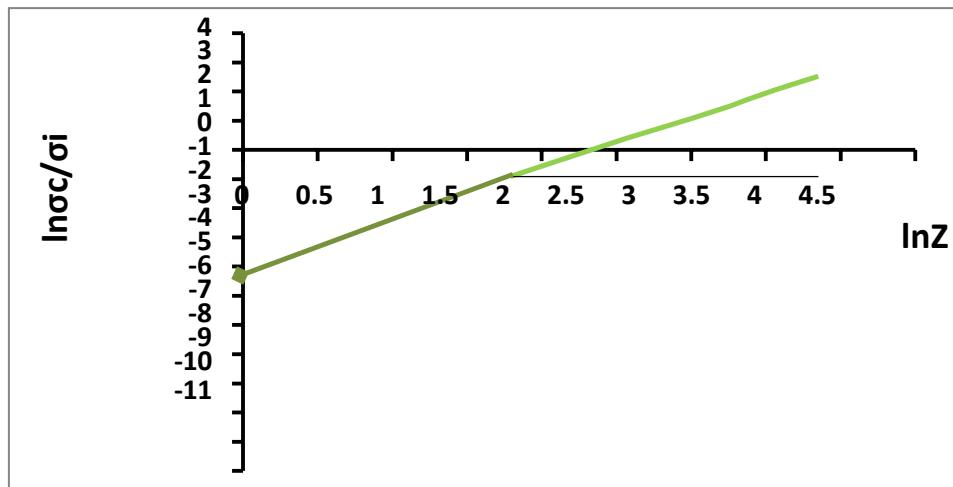
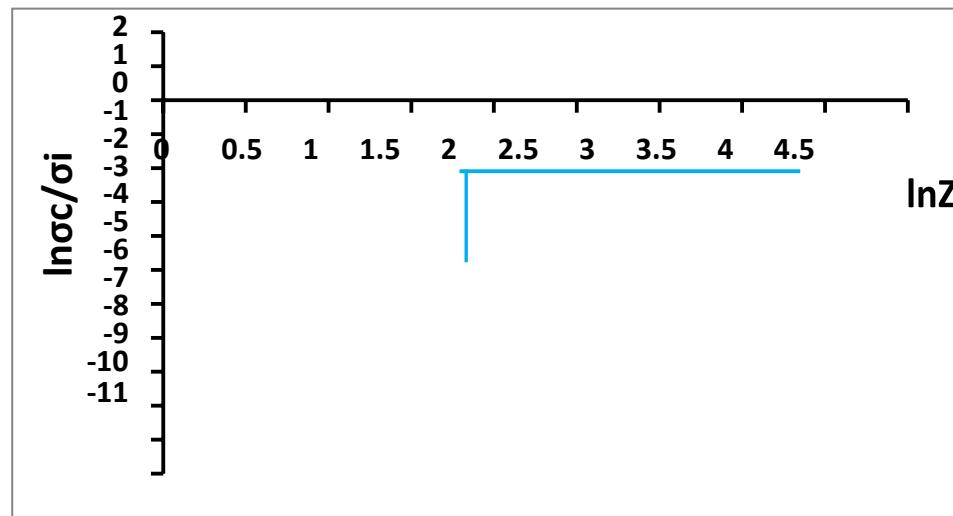
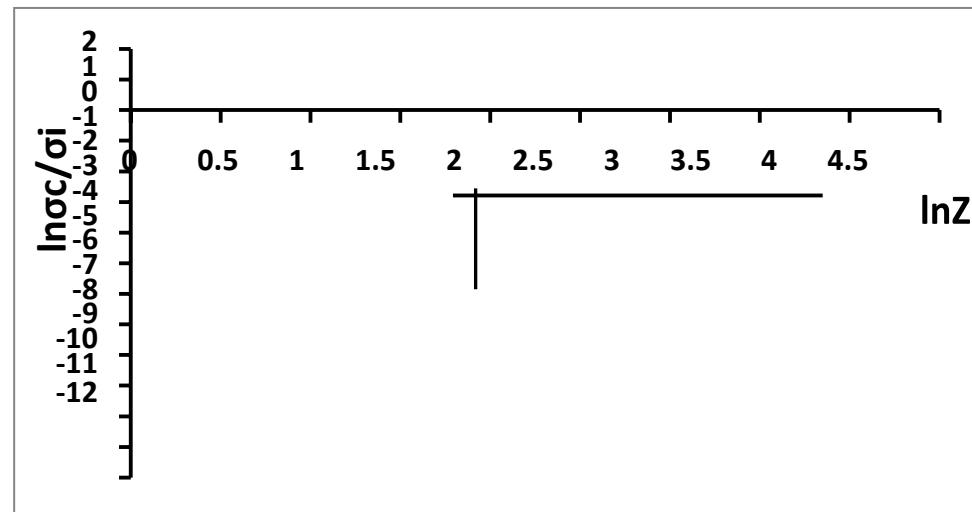
Z	$\ln Z$	σ_c	σ_i	$\frac{\sigma_c}{\sigma_i}$	$\frac{\sigma_c}{\sigma_i} \ln \frac{\sigma_c}{\sigma_i}$
6	1.7917	0.00215	1.6	0.01344	-6.6123
13	2.5649	0.0177	3.47	0.005101	-5.2783
20	2.9957	0.0529	5.33	0.009925	-4.6127
26	3.258	0.103	6.92	0.01488	-4.2074
29	3.367	0.138	7.72	0.01788	-4.0243
35	3.555	0.225	9.31	0.02417	-3.7227
42	3.737	0.336	11.2	0.03241	-3.4293
47	3.85	0.488	12.5	0.0523904	-3.2432

Table 12-Values of the atomic cross-sections σ_c and σ_i for an energy value of E=800kev [11, 13].

Z	$\ln Z$	σ_c	σ_i	$\frac{\sigma_c}{\sigma_i}$	$\frac{\sigma_c}{\sigma_i} \ln \frac{\sigma_c}{\sigma_i}$
6	1.7917	0.00121	1.41	0.0008582	-7.0607
13	2.5649	0.00999	3.5	0.003275	-5.7213
20	2.9957	0.0298	4.69	0.006354	-5.0587
26	3.258	0.0580	6.09	0.009524	-4.6540
29	3.367	0.0776	6.79	0.01143	-4.4716
35	3.555	0.127	8.19	0.01551	-4.1665
42	3.737	0.204	9.82	0.02077	-3.8741
47	3.85	0.275	11.0	0.025	-3.6889

Results

Plots of $\ln(\sigma_c/\sigma_i)$ against $\ln z$ are shown in Figures (1-12) for photon energies of 20-800 keV, respectively, given as a straight line whose slope is $(n-1)$ and y-intercept is $(\ln k)$. This was performed for all the values of energies limited by the range of atomic number stated above.

Figure 1- A plot of $\ln(\sigma_c/\sigma_i)$ against $\ln z$ for photon energy of 20 keV.Figure 2-A plot of $\ln(\sigma_c/\sigma_i)$ against $\ln z$ for photon energy of 40 keV.Figure 3-A plot of $\ln(\sigma_c/\sigma_i)$ against $\ln z$ for photon energy of 60 keV.

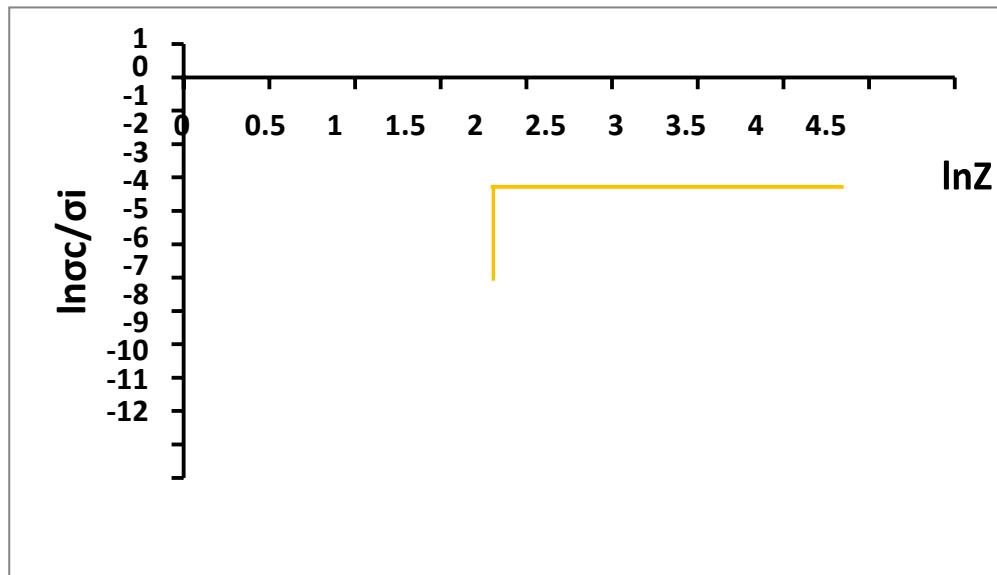


Figure 4-A plot of $\ln(\sigma_c/\sigma_i)$ against $\ln z$ for photon energy of 80 keV.

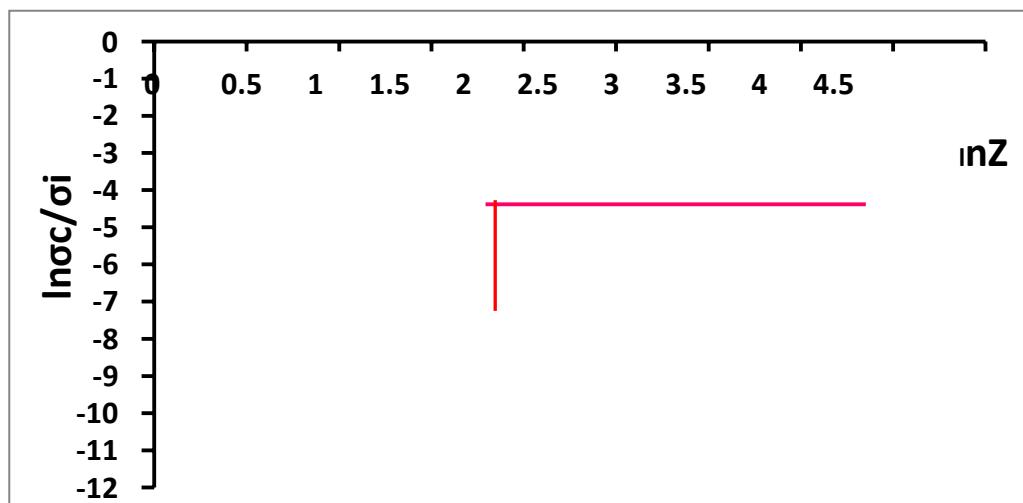


Figure 5-A plot of $\ln(\sigma_c/\sigma_i)$ against $\ln z$ for photon energy of 100 keV.

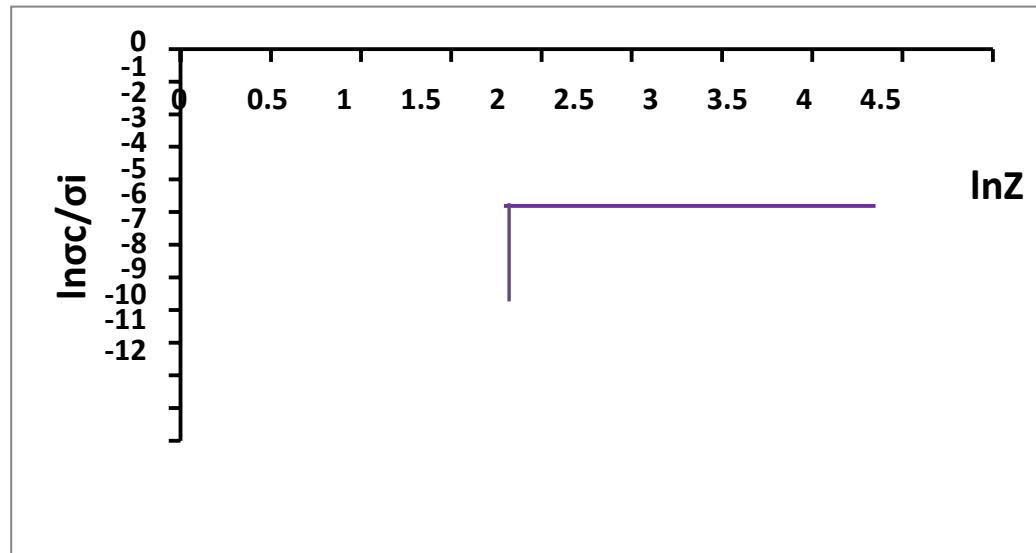


Figure 6-A plot of $\ln(\sigma_c/\sigma_i)$ against $\ln z$ for photon energy of 150 keV.

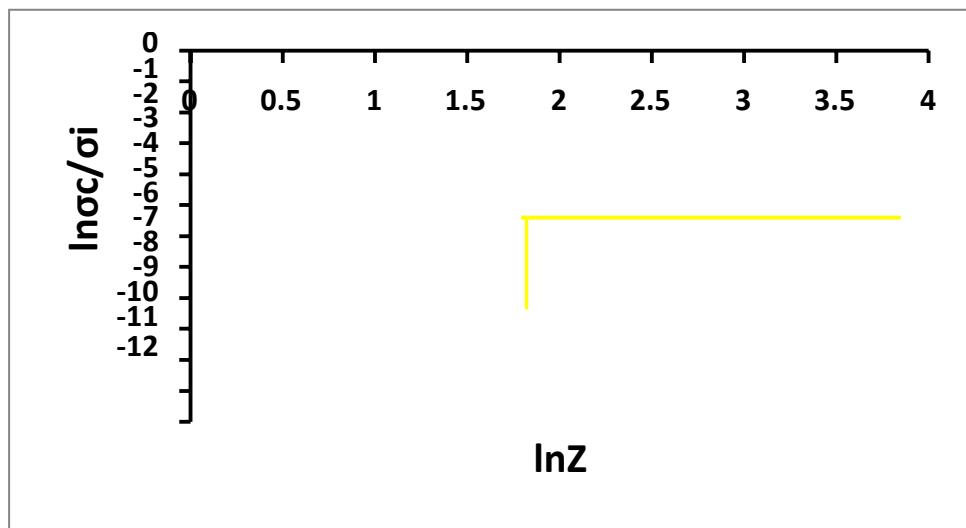


Figure 7-A plot of $\ln(\sigma_c/\sigma_i)$ against $\ln z$ for photon energy of 200 keV.

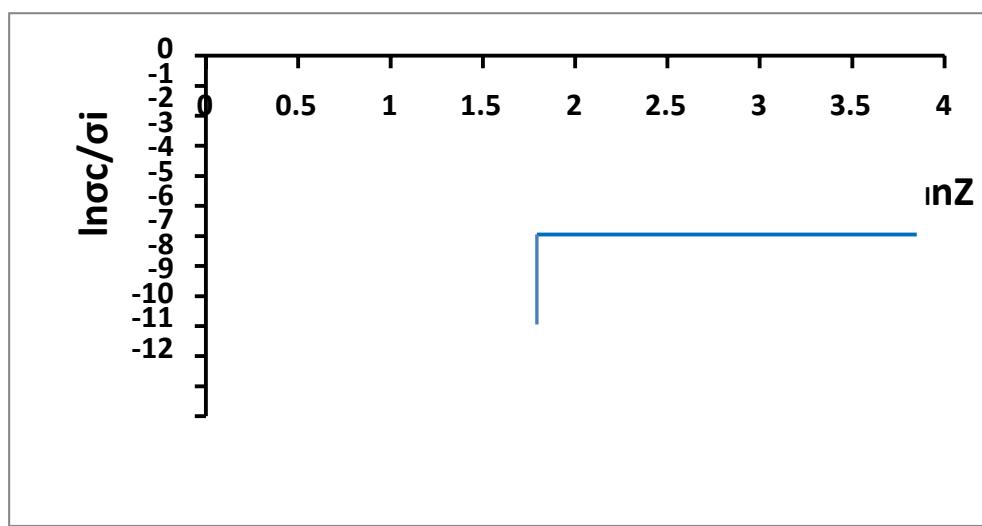


Figure 8-A plot of $\ln(\sigma_c/\sigma_i)$ against $\ln z$ for photon energy of 300 keV.

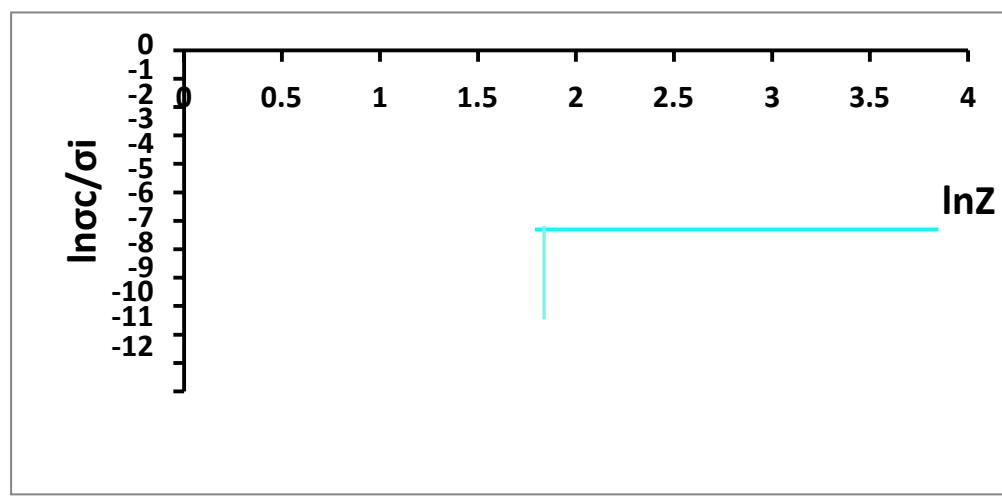


Figure 9-A plot of $\ln(\sigma_c/\sigma_i)$ against $\ln z$ for photon energy of 400 keV.

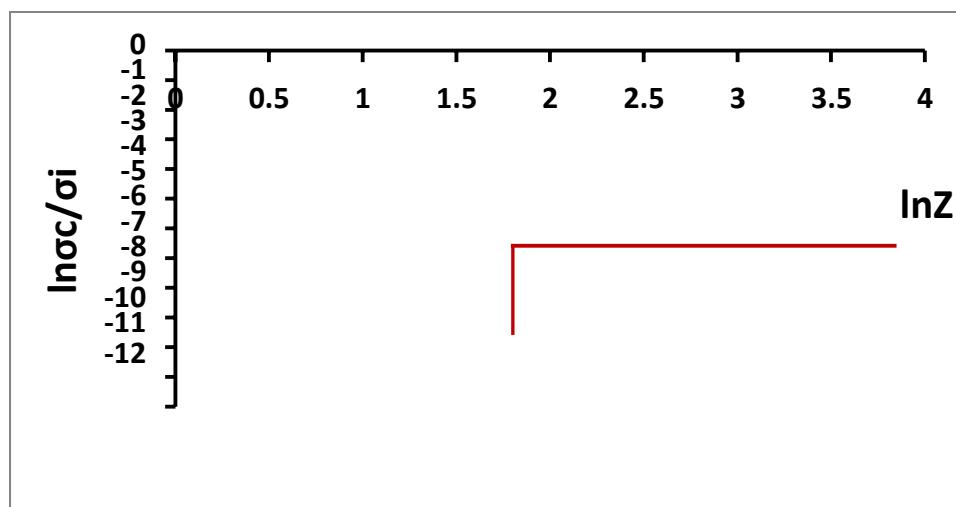


Figure 10-A plot of $\ln(\sigma_c/\sigma_i)$ against $\ln z$ for photon energy of 500 keV.

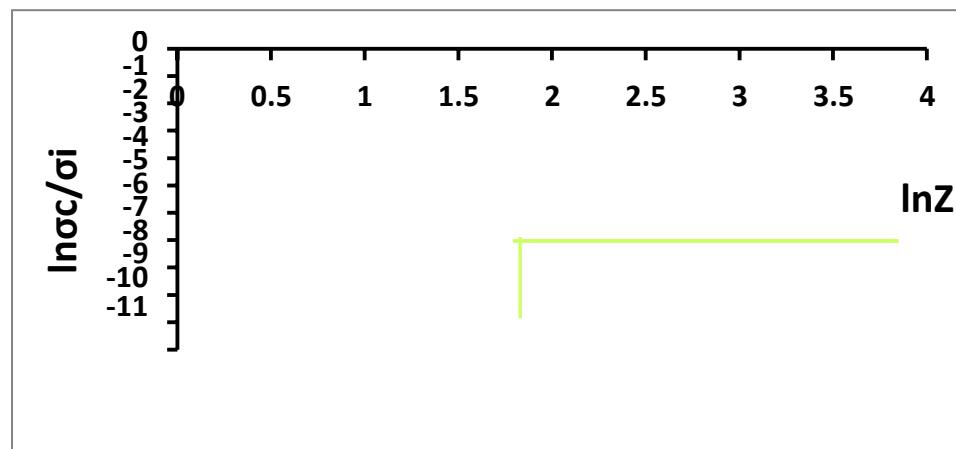


Figure 11- A plot of $\ln(\sigma_c/\sigma_i)$ against $\ln z$ for photon energy of 600 keV.

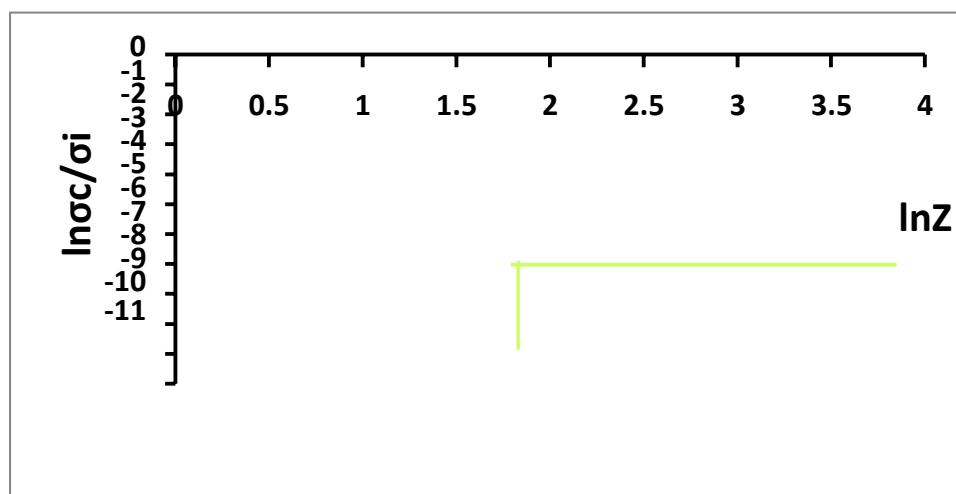


Figure 12-A plot of $\ln(\sigma_c/\sigma_i)$ against $\ln z$ for photon energy of 800 keV.

The energy dependent constant K was obtained using Microsoft Excel for tables (1-12) and graphs (1-12) for each of the different energy values (20-800 Kev). Table (13) and graph (13) which were obtained by using auto-cad program show the exponential change between the different energies with the energy dependent constant K .

Table 13-Values of different energy with the energy dependent constant K.

E(kev)	<i>slope = n - 1</i>	$\ln K$	$K \times 10^{-4}$
20	1.6574	-3.878	206.92
40	1.645	-5.037	64.93
60	1.6433	-5.7268	32.6
80	1.6443	-6.222	19.85
100	1.6524	-6.6179	13.36
150	1.6512	-7.2983	6.767
200	1.6741	-7.73.85	4.528
300	1.6845	-8.516	2.00
400	1.6874	-8.98597.3	1.259
500	1.6904	-9.3493	0.8703
600	1.6924	-9.641	0.650
800	1.6943	-10.0905	0.415

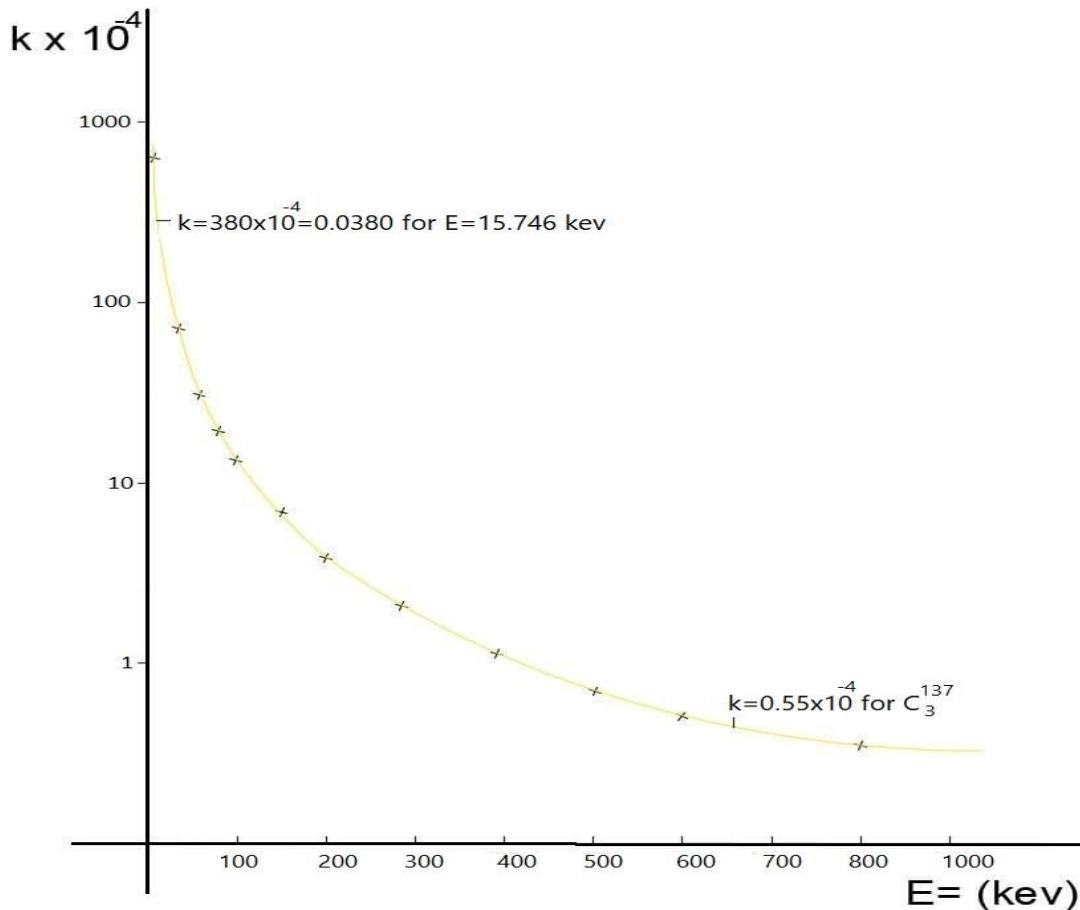


Figure 13-The relationship between the photon energies E and the energy dependent constant K.

Discussion and Conclusions

The dependence of the cross-section of the coherent and in-coherent radiation peaks in the x-ray absorption experiment of different energies (20-800 Kev) on the value of the atomic number was included based on the published data for (8) elements, ranging from (C-Ag). The obtained value of the energy dependent constant is not fixed for all elements, but rather changes exponentially with the value of photon energy . Therefore, we conclude that the energy dependent constant depends on the photon energies [14,15].

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