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A New Method for Solving Fully Fuzzy Multi-Objective Linear Programming Problems

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Abstract
In this paper we present a new method for solving fully fuzzy multi-objective linear programming problems and find the fuzzy optimal solution of it. Numerical examples are provided to illustrate the method.

Keywords: fully fuzzy multi-objective programming, fully fuzzy linear programming number , triangular fuzzy number .

طريقة جديدة لحل مسائل البرمجة المتعددة الأهداف الخطية الضبابية بصورة كاملة

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الخلاصة :
في هذا البحث سنقدم طريقة جديدة لحل مسائل البرمجة المتعددة الأهداف الخطية الضبابية بصورة كاملة وإيجاد الحل الأمثل الضبابي. والأمثلة العددية جهزت لتوضيح الطريقة .

1-Introduction

Fuzzy set theory has been applied to many disciplines such as management sciences, mathematical modeling , control theory and industrial applications. Zadeh [1] introduced the fuzzy set theory to deal with uncertainty due to imprecision and vagueness . S.Mohammed [2] proposed a novel project scheduling method based on fully fuzzy linear programming. A.O.Hamadameen and Z.M.Zainuddin [3] used the fuzzy stochastic linear programming problems with uncertainty probability distribution. M.M.Shamooshaki , A.Hosseinzadeh and S.A.Edalatpanah [4] presented a new method for solving fully fuzzy linear programming with LR-type fuzzy numbers . M.Mehdi , A.Hosseinzadeh and S.Ahmed [5] proposed a new method for solving fully fuzzy linear programming problems by using the lexicography method . So P.Pandit [6] introduce multi-objective linear programming problems involving fuzzy parameters. Also A.Chaudhuri and K.De [7] used fuzzy multi-objective linear programming for traveling salesman problem .

In this paper we proposed a new method for solving fully fuzzy multi-objective linear programming problems and find the fuzzy optimal solution . A new method is illustrated with the help of numerical examples .

2- Preliminaries

2.1- Basic Definitions

We first review some known definitions which relevant to this work .

Definition 2.1 [8] : if X is a collection of objects denoted generically by x , then a fuzzy set A in X is defined to be a set of ordered pairs A={ $(x , \mu_A(x))$: $x \in X$} , where $\mu_A(x)$ is called the membership

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function for the fuzzy set . The membership function maps each element of X to a membership value between 0 and 1 . We assume that X is the real line R .

Definition 2.2 [9] : A fuzzy number $\tilde{a}$ is a triangular fuzzy number denoted by (a_1 , a_2 , a_3) where a_1, a_2 and a_3 are real numbers and its membership function $\mu_{\tilde{a}}(x)$ is given below :

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & for \quad a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-a_2} & for \quad a_2 \le x \le a_3 \\ 0 & other\ wise \end{cases}$$

Definition 2.3 [10] : Let (a_1 , a_2 , a_3) and (b_1 , b_2 , b_3) be two triangular fuzzy numbers .Then

(i) $(a_1 , a_2 , a_3) (+) (b_1 , b_2 , b_3 = (a_1 + b_1 , a_2 + b_2 , a_3 + b_3)$.

(ii) $(a_1 , a_2 , a_3) (-) (b_1 , b_2 , b_3) = (a_1 - b_3 , a_2 - b_2 , a_3 - b_1)$.`

(iii) $k(a1,a2,a3) = (ka1,ka2,ka3)$, *for* $k \ge 0$.

(iv) $k(a1,a2,a3) = (ka3,ka2,ka1)$, *for* $k < 0$

Let $F(R)$ be the set of all real triangular fuzzy numbers.

Definition 2.4 [10] : Let $\tilde{A}$= (a_1 , a_2 , a_3) be in $F(R)$. Then

(i) $\tilde{A}= \tilde{B} \Leftrightarrow a i = b i$, for all *for* i =1 to 3 and

(ii) $\tilde{A} \le \tilde{B} \Leftrightarrow a i \le b i$, for all *for* i =1 to 3 .

2.2- Fully fuzzy linear programming problem [11 , 12]

Fully fuzzy linear programming problem can be written :

(Q) max (or min) $\tilde{Z} = (\tilde{C}^t \otimes \tilde{X})$

Subject to

$$\tilde{A} \otimes \tilde{X} = \tilde{b}$$

$\tilde{X}$: is non- negative fuzzy number ,

where $\tilde{C}^t = [\tilde{C}_j]_{1\times n}$, $\tilde{X} = [\tilde{X}_j]_{n\times 1}$, $\tilde{A} = [\tilde{a}_{ij}]_{m\times n}$, $\tilde{b} = [\tilde{b}_i]_{m\times 1}$ and $\tilde{a}_{ij}$, $\tilde{C}_j$, $\tilde{X}_j$, $\tilde{b}_i \in F(R)$.

2.3- Fully fuzzy multi-objective linear programming problems [13 , 14]

Let the parameters $\tilde{Z}$, $\tilde{a}_{ij}$, $\tilde{C}_j$, $\tilde{X}_j$ and $\tilde{b}_i$ be the triangular fuzzy numbers (Z_1 , Z_2 , Z_3) , (p_j , q_j , r_j) , (x_j , y_j , t_j) , $(a_{ij} , b_{ij} , c_{ij})$ and (b_j , g_j , h_j) respectively – then , the problem (Q) can be written as follows :

(Q) Maximize $(Z_1 , Z_2 , Z_3) \approx \sum_{j=1}^{n} (p_j , q_j , r_j) \otimes (x_j , y_j , t_j)$

Subject to

$\sum_{j=1}^{n} (a_{ij} , b_{ij} , c_{ij}) \otimes (x_j , y_j , t_j) \{ \le , \approx , \ge \} (b_j , g_j , h_j)$ for all i=1,2,….,m

$(x_j , y_j , t_j) \ge \tilde{0}$, j =1,2,…,m.

Now, using the arithmetic operations and partial ordering relations, we write the given FLPP as a MOLP problem which is given below:

(M) Maximize $z_1 = \sum_{j=1}^{n} lower\ value\ of\ ((p_j , q_j , r_j) \otimes (x_j , y_j , t_j))$

Maximize $z_2 = \sum_{j=1}^{n} middle\ value\ of\ ((p_j , q_j , r_j) \otimes (x_j , y_j , t_j))$

Maximize $z_3 = \sum_{j=1}^{n} upper\ value\ of\ ((p_j , q_j , r_j) \otimes (x_j , y_j , t_j))$

Subject to

$\sum_{j=1}^{n} lower\ value\ of\ ((a_{ij} , b_{ij} , c_{ij}) \otimes (x_j , y_j , t_j)) \{ \le , = , \ge \}$ b_i ,
for all i=1,2,….,m ;

$\sum_{j=1}^{n} middle\ value\ of\ ((a_{ij} , b_{ij} , c_{ij}) \otimes (x_j , y_j , t_j)) \{ \le , = , \ge \}$ g_i ,
for all i=1,2,….,m ;

$\sum_{j=1}^{n} upper\ value\ of\ ((a_{ij} , b_{ij} , c_{ij}) \otimes (x_j , y_j , t_j)) \{ \le , = , \ge \}$ h_i ,
for all i=1,2,….,m ;

And $x_j \le y_j \le t_j$, j =1,2,…,m. and all decision variables are non-negative .

3-The proposed method for solving fully fuzzy multi-objective linear programming problems

In this study ,we proposed a new method to convert fully fuzzy multi-objective linear programming to fully fuzzy linear programming problem and we get the optimal fuzzy solution .

Step 1 : by the weighting problem of fully fuzzy multi-objective linear programming is converted to the fully fuzzy objective linear problem .

The weighting problem of fully fuzzy multi-objective linear programming:

Maximize (Minimize)

$(\sum_{r=1}^{k} w^r ((\tilde{c}^r)^t \otimes \tilde{X}) = w^1 ((\tilde{c}^1)^t \otimes \tilde{X}) \oplus w^2 ((\tilde{c}^2)^t \otimes \tilde{X}) \oplus ... \oplus w^k ((\tilde{c}^k)^t \otimes \tilde{X}))$,
Subject to
$\tilde{A} \otimes \tilde{X} = \tilde{b}$
$\tilde{X} \geq 0$, $\sum_{r=1}^{k} w^r = 1$, $w^r \geq 0$.
fully fuzzy multi-objective linear programming is converted to the fully fuzzy objective linear problem:
Maximize (Minimize) $(\tilde{c})^t \otimes \tilde{X}$,
Subject to
$\tilde{A} \otimes \tilde{X} = \tilde{b}$, $\tilde{X} \geq 0$.
Step 2 : construct (MLP),(ULP) and (LLP) problem for the given (FLP) problem .
Step 3 : solve the (MLP) problem and then the (LLP) problem by using simplex method and find the optimal solution and obtain the values of all variables x_j, y_j and t_j and values of all objectives Z_1, Z_2 and Z_3.

4- Numerical Examples

The proposed method is illustrated by the following examples .
Example 4.1
Maximize ((3,5,7) $\otimes \tilde{X}_1 \oplus$(2,4,8) $\otimes \tilde{X}_2$) , ((3,5,10) $\otimes \tilde{X}_1 \oplus$(1,7,8) $\otimes \tilde{X}_2$)
Subject to (4,5,9) $\otimes \tilde{X}_1 \oplus$ (2,7,8) $\otimes \tilde{X}_2$ = (4,10,20)
(0,3,7) $\otimes \tilde{X}_1 \oplus$ (1,2,10) $\otimes \tilde{X}_2$ = (2,5,18)
$\tilde{X}_1$, $\tilde{X}_2 \geq 0$
By step 1: Assume w=(0.5,0.5) , then obtain to FFMLP problem can be written as :
Maximize ((1.5,2.5,3.5)$\otimes \tilde{X}_1 \oplus$(1,2,4) $\otimes \tilde{X}_2$),((1.5,2.5,5)$\otimes \tilde{X}_1 \oplus$(0.5,3.5,4) $\otimes \tilde{X}_2$)
Maximize ((3,5,8.5)$\otimes \tilde{X}_1 \oplus$(1.5,5.5,8) $\otimes \tilde{X}_2$)
Subject to (4,5,9) $\otimes \tilde{X}_1 \oplus$ (2,7,8) $\otimes \tilde{X}_2$ = (4,10,20)
(0,3,7) $\otimes \tilde{X}_1 \oplus$ (1,2,10) $\otimes \tilde{X}_2$ = (2,5,18)
$\tilde{X}_1$, $\tilde{X}_2 \geq 0$
Assume $\tilde{X}_1 = (x_1, y_1, t_1)$, $\tilde{X}_2 = (x_2, y_2, t_2)$ and $\tilde{Z} = (Z_1, Z_2, Z_3)$
We get to the problem FLPP in the following :
Maximize $Z_1 = 3t_1 + 1.5 t_2$
Maximize $Z_2 = 5y_1 + 5.5 y_2$
Maximize $Z_3 = 8.5t_1 + 8 t_2$
Subject to
$4x_1 + 2x_2 = 4$; $0x_1 + 1x_2 = 2$;
$5y_1 + 7y_2 = 10$; $3y_1 + 2y_2 = 5$;
$9t_1 + 8 t_2 = 20$; $7t_1 + 10 t_2 = 18$;
$x_1, x_2 \geq 0$, $y_1, y_2 \geq 0$, $t_1, t_2 \geq 0$.
By step 2 : we get to the middle level problem :
(P_2) : Maximize $Z_2 = 5y_1 + 5.5 y_2$
Subject to
$5y_1 + 7y_2 = 10$; $3y_1 + 2y_2 = 5$;
$y_1, y_2 \geq 0$
Using simplex method to solve the problem (P_2) , we get to the optimal solution
$y_1 = 1.3636$; $y_2 = 0.4545$ and $Z_2 = 9.3182$
the upper level problem in the following :
(P_3) : Maximize $Z_3 = 8.5t_1 + 8 t_2$
Subject to
$8.5t_1 + 8t_2 \geq 9.3182$; $9t_1 + 8 t_2 = 20$; $7t_1 + 10 t_2 = 18$;
$t_1 \geq y_1$, $t_2 \geq y_2$, , $t_1, t_2 \geq 0$.
Using simplex method to solve the problem (P_3) when $y_1 = 1.3636$; $y_2 = 0.4545$, we get to the optimal solution $t_1 = 1.6471$, $t_2 = 0.6471$ and $Z_2 = 19.1765$.
the lower level problem in the following :
(P_1) : Maximize $Z_1 = 3t_1 + 1.5 t_2$
Subject to

$3\,t_1 + 1.5\,t_2 \le 9.3182$; $4\,x_1 + 2\ x_2 = 4$; $0\,x_1 + 1\ x_2 = 2$;
$x_1 \le y_1$, $x_2 \le y_2$; x_1 , $x_2 \ge 0$.
Using simplex method to solve the problem (P1) when $t_1 = 3.1061$, $t_2 = 0$, y_1=1.3636 ; $y_2 = 0.4545$, we get to the optimal solution x_1= 0.7728 , x_2 = 0.4545 and Z_1= 9.3182 .
Now the optimal fuzzy solution to the given fully fuzzy linear programming problem is : $\tilde{X}_1 =$ (0.7728 , 1.3636 , 1.6471) , $\tilde{X}_2 =$ (0.4545 , 0.4545 , 0.6471) and $\tilde{Z}$= (9.3182 , 9.3182 , 19.1765) .

Example 4.2

Maximize ((1,2,3) $\otimes$ $\tilde{X}_1$ $\oplus$(2,4,5) $\otimes$ $\tilde{X}_2$) , ((2,3,4) $\otimes$ $\tilde{X}_1$ $\oplus$(3,4,5) $\otimes$ $\tilde{X}_2$)
Subject to (0,1,2) $\otimes$ $\tilde{X}_1$ $\oplus$ (1,2,3) $\otimes$ $\tilde{X}_2$ = (1,10,27)
(1,2,3) $\otimes$ $\tilde{X}_1$ $\oplus$ (0,1,2) $\otimes$ $\tilde{X}_2$ = (2,11,28)
$\tilde{X}_1$, $\tilde{X}_2 \ge 0$

By step 1: Assume w=(0.5,0.5) , then obtain to FFMLP problem can be written as :
Maximize ((0.5,1,1.5)$\otimes$ $\tilde{X}_1$ $\oplus$(1,2,2.5) $\otimes$ $\tilde{X}_2$),((1,1.5,2)$\otimes$ $\tilde{X}_1$ $\oplus$(1.5,2,2.5) $\otimes$ $\tilde{X}_2$)
Maximize ((1.5,2.5,3.5)$\otimes$ $\tilde{X}_1$ $\oplus$(2.5,4,5) $\otimes$ $\tilde{X}_2$)
Subject to (0,1,2) $\otimes$ $\tilde{X}_1$ $\oplus$ (1,2,3) $\otimes$ $\tilde{X}_2$ = (1,10,27)
(1,2,3) $\otimes$ $\tilde{X}_1$ $\oplus$ (0,1,2) $\otimes$ $\tilde{X}_2$ = (2,11,28)
$\tilde{X}_1$, $\tilde{X}_2 \ge 0$

Assume $\tilde{X}_1 = (x_1 , y_1 , t_1)$, $\tilde{X}_2 = (x_2 , y_2 , t_2)$ and $\tilde{Z}$= (Z_1 , Z_2 , Z_3)
We get to the problem FLPP in the following :
Maximize Z_1 = 1.5 t_1 + 2.5 t_2
Maximize Z_2 = 2.5 y_1 + 4 y_2
Maximize Z_3 = 3.5 t_1 + 5 t_2
Subject to
$0\,x_1 + 1\,x_2 = 1$; $1\,x_1 + 0\,x_2 = 2$;
$1\,y_1 + 2\,y_2 = 10$; $2\,y_1 + 1\,y_2 = 11$;
$2\,t_1 + 3\,t_2 = 27$; $3\,t_1 + 2\ t_2 = 28$;
x_1 , $x_2 \ge 0$, y_1 , $y_2 \ge 0$, t_1 , $t_2 \ge 0$.

By step 2 : we get to the middle level problem :
(P2) : Maximize Z_2 = 2.5 y_1 + 4 y_2
Subject to
$1\,y_1 + 2\,y_2 = 10$; $2\,y_1 + 1\,y_2 = 11$;
y_1 , $y_2 \ge 0$
Using simplex method to solve the problem (P2) , we get to the optimal solution
y_1 = 4 ; $y_2 = 3$ and Z_2= 22
the upper level problem in the following :
(P3) : Maximize Z_3 = 3.5 t_1 + 5 t_2
Subject to
$3.5\,t_1 + 5\,t_2 \ge 22$; $2\,t_1 + 3\,t_2 = 27$; $3\,t_1 + 2\,t_2 = 28$;
$t_1 \ge y_1$, $t_2 \ge y_2$, , t_1 , $t_2 \ge 0$.
Using simplex method to solve the problem (P3) when y_1 = 4 ; $y_2 = 3$, we get to the optimal solution t_1= 6 , t_2 = 5 and Z_2= 46 .
the lower level problem in the following :
(P1) : Maximize Z_1 = 1.5 t_1 + 2.5 t_2
Subject to
$1.5\,t_1 + 2.5\,t_2 \le 22$; $0\,x_1 + 1\,x_2 = 1$; $1\,x_1 + 0\,x_2 = 2$;
$x_1 \le y_1$, $x_2 \le y_2$; x_1 , $x_2 \ge 0$.
Using simplex method to solve the problem (P1) when $t_1 = 0$, $t_2 = 8$, y_1= 4 ; $y_2 = 3$, we get to the optimal solution $x_1 = 2$, x_2 = 1 and Z_1= 22 .
Now the optimal fuzzy solution to the given fully fuzzy linear programming problem is : $\tilde{X}_1 =$ (2 , 4 , 6) , $\tilde{X}_2 =$ (1 , 3 , 5) and $\tilde{Z}$= (22, 22 , 46) .

5- Conclusion

In this paper, a new method was proposed for solving fully fuzzy multi-objective linear programming problems when the variables triangular fuzzy numbers and to find an optimal fuzzy solution to a fuzzy linear programming.

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