DOI: 10.24996/ijs.2023.64.4.27

ISSN: 0067-2904

Doubly Type II Censoring of Two Stress-Strength System Reliability Estimation for Generalized Exponential-Poisson Distribution

Hajer M AbdAwon*, Nada S Karam
Department of Mathematics, College of Education, Mustansiriyah University-Baghdad-Iraq

Received: 31/3/2022 Accepted: 6/8/2022 Published: 30/4/2023

Abstract

In this paper, a Bayesian analysis is made to estimate the Reliability of two stress-strength model systems. First: the reliability R_1 of a one component strengths X under stress Y. Second, reliability R_2 of one component strength under three stresses. Where X and Y are independent generalized exponential-Poison random variables with parameters (α,λ,θ) and (β,λ,θ) . The analysis is concerned with and based on doubly type II censored samples using gamma prior under four different loss functions, namely quadratic loss function, weighted loss functions, linear and non-linear exponential loss function. The estimators are compared by mean squared error criteria due to a simulation study. We also find that the mean square error is the best performance of the estimator from that found in quadratic, weighted, linear and non-linear exponential loss functions.

Keywords: Bayesian Analyses, Generalized Exponential Poisson distribution, Doubly type II censored sample stress -strength , Quadratic loss function, Weighted loss function, Linear and non-linear exponential loss functions.

تقدير موثوقية نظامي الاجهاد والمتانة لبيانات المراقبة من النوع الثاني المضاعف لتوزيع بواسون الاسي المعمم

هاجر محمد عبد عون* ، ندى صباح كرم
قسم الرياضيات، كلية التربيه، الجامعه المستنصرية، بغداد، العراق

الخلاصة

في هذا البحث تم اجراء تحليل بايزي لتقدير موثوقية انظمة نموذجين الاول R1 بتسليط مكونة واحدة Xعلى الاجهاد Yوالثاني بتسليط مكونة على ثلاث اجهادات (ضغوط)حيث X,Y متغيرات عشوائية مستقلة تم تقدير التحليل المعني على اساس عينات ذات رقابة من النوع الثاني المضاعف باستخدام دالة كاما السابقة تحت اربع دوال خسارة هي دالة الخطأ التربيعية ودالة الوزن ودالة الخطأ الاسي الخطي ودالة الخطأ الاسي غير الخطي حيث تم مقارنة التقدير عن طريق متوسط مربعات الخطا تم اجراء محاكاة للعثور على افضل اداء وقد بينت الدراسة ان الافضل هي دالة الخطأ التربيعي

1. Introduction

In 1998, Adamidis and Loukas introduced a two-parameter distribution with decreasing failure rate [1]. This distribution is known as exponential-geometric distribution. Kus (2007)

*Email: hajrm9990@gmail.com

[2]. introduced in the same shape a two-parameter distribution known as exponential distribution with the Poisson distribution. In (2009) [3] Barreto- Souza and Cribari-Neto generalized exponential-Poisson with decreasing or increasing failure rate. The two parameter exponential-Poisson (EP) with cumulative distribution function (c.d.f) is given as follows:[4]

$$F(x) = \frac{1 - exp\left[-\lambda\left(1 - exp(-\theta x)\right)\right]}{1 - exp(-\lambda)} \qquad x > 0, \theta, \lambda > 0$$

The Generalized Exponential–Poisson distribution (GEP) for the random variable X with parameters (α,λ,θ) is given as follows:[3]

$$F(x) = \left[\frac{1 - exp\left(-\lambda\left(1 - exp(-\theta x)\right)\right)}{1 - exp(-\lambda)}\right]^{\alpha} = \left[\frac{1 - A_x}{k}\right]^{\alpha}. \tag{1}$$

Where $A_x = e^{-\lambda\left(1-e^{-\theta x}\right)}$ for $\alpha > 0$ and $k = 1 - e^{-\lambda}$.when α is the shape parameter and (θ, λ) represents the scale parameters .The corresponding probability density function (p.d.f) is defined as follows:

$$f(x) = \frac{\alpha\lambda\theta}{(1 - e^{-\lambda})^{\alpha}}\left[1 - e^{-\lambda\left(1-e^{-\theta x}\right)}\right]^{\alpha-1} e^{-\theta x} e^{-\lambda\left(1-e^{-\theta x}\right)} \tag{2}$$

$$f(x) = \frac{\alpha\lambda\theta}{k^{\alpha}}\, Ax\, [1 - A_x]^{\alpha-1} e^{-\theta x}$$

The paper is organized as in section 2, the general expression of $R_1$and R_2 are given for one stress-strength and one strength composed under three stresses. The Bayesian estimators are found for R_1 and R_2 under four different loss functions are given in section 3. Finally, in section 4 the performance of the estimators is illustrating by experiment simulation study.

2. Reliability of the systems for GEP Stress-Strength Models

The purpose of this section is to obtain the reliabilities expression of two different systems for stress- strength models.

2.1 One component system reliability.

The reliability R_1 of a component operating under stress –strength system given by [5]

$$R_1 = P(Y < X) \tag{3}$$

Let the strength random variable $X \sim GEP(\alpha, \lambda, \theta)$ and the stress random variable $Y \sim GEP(\beta, \lambda, \theta)$ when X and Y are independent but not identical.then the (c.d.f) for Y can be written as follows:

$$G(y) = \left[\frac{1 - exp\left[-\lambda\left(1 - exp(-\theta y)\right)\right]}{1 - exp(-\lambda)}\right]^{\beta} = \left[\frac{1 - A_y}{k}\right]^{\beta}$$

Now, the reliabilityR_1 from equation (3) can be given as follows:

$$R_1 = \int_0^{\infty} G_y(x)\, f(x)\, dx \;\; = \int_0^{\infty} \left(\frac{1-A_x}{k}\right)^{\beta} \frac{\alpha\lambda\theta}{k^{\alpha}} A_x e^{-\theta x}(1 - A_x)^{\alpha-1} dx.$$

$$= \frac{\alpha\lambda\theta}{k^{\beta+\alpha}} \int_0^{\infty} (1 - A_x)^{\beta+\alpha-1} A_x e^{-\theta x} dx$$

Because $\int_0^{\infty} f(x)dx = 1$, So that from equation (2), then we can use the following:

$$\int_0^{\infty} (1 - A_x)^{\alpha-1} A_x\, e^{-\theta x} dx = \frac{k^{\alpha}}{\alpha\lambda\theta} \;\; . \tag{4}$$

So we get

$$R_1 = \frac{\alpha\lambda\theta}{k^{\beta+\alpha}} \int_0^\infty e^{-\theta x} A_x (1 - A_x)^{\beta+\alpha-1} dx = \frac{\alpha}{\alpha+\beta} \tag{5}$$

2.2 Three stress-one strength

The reliability of a component that has X strength is exposed to three independent stresses namely Y_1, Y_2 and Y_3. The stress-strength reliability can be obtained as follows: $R_2 = P(max(Y_1, Y_2, Y_3) < X)$.

Let $X \sim GEP(\alpha, \lambda, \theta)$ be strength random variable and $Y_1 \sim GEP(\beta_1, \lambda, \theta)$, $Y_2 \sim GEP(\beta_2, \lambda, \theta)$ and $Y_3 \sim GEP(\beta_3, \lambda, \theta)$are the stresses random variables, then the reliability can be obtained as follows [6]:

$$R_2 = \int_{x=0}^{\infty} P(Y_1 < X) P(Y_2 < X) P(Y_3 < X)\, f(x) dx$$

Since X, Y_1, Y_2 and Y_3 are non-identical independently distributed, we can get:

$$R_2 = \int_{x=0}^{\infty} G_{y_1}(x) G_{y_2}(x) G_{y_3}(x)\, f(x) dx$$

$$R_2 = \int_{x=0}^{\infty} \left(\frac{1 - A_x}{k}\right)^{\beta_1} \left(\frac{1 - A_x}{k}\right)^{\beta_2} \left(\frac{1 - A_x}{k}\right)^{\beta_3} \frac{\alpha\lambda\theta}{k^\alpha} A_x e^{-\theta x} (1 - A_x)^{\alpha-1} dx$$

$$R_2 = \frac{\alpha\lambda\theta}{k^{\beta_1+\beta_2+\beta_3+\alpha}} \int_{x=0}^{\infty} (1 - A_x)^{\beta_1+\beta_2+\beta_3+\alpha-1} A_x\, e^{-\theta x} dx$$

By using equation (4) we get

$$R_2 = \frac{\alpha}{\alpha + \beta_1 + \beta_2 + \beta_3} \tag{6}$$

3. Bayes analysis

In this section, the Bayes estimators of reliabilities R_1 and R_2 are given based on a doubly type II censored sample using gamma prior under quadratic, weighted, linear and non-linear exponential loss functions.

3.1 Doubly type II censored sample

When we have n units that are subject to testing and we want to censor the work of m units, where m=s-r+1 and r $< s < n$,any censoring data $(x_r, \dots, x_s)$,so in this case ,it cannot be determined the random variable in the time and thus stop the test up to get m units of censored and upon arrival to the s , The likelihood function for this type of data[7]:

Such that $x_{(1)} < x_{(2)} < \cdots < x_{(r-1)} < x_{(r)} < x_{(r+1)} < \cdots < x_{(s)} < x_{(s+1)} < \cdots < x_{(n)}$

$$L(\alpha|x) = \left(\frac{n!}{(r-1)!\,(n-s)!}\right) \left(F(x_r)\right)^{r-1} \left(1 - F(x_s)\right)^{n-s} \prod_{i=r}^{s} f(x_i, \alpha) \tag{7}$$

$$\prod_{i=r}^{s} f(x_i, \alpha) = \prod_{i=r}^{s} \left(\frac{\alpha\lambda\theta}{k^\alpha}\, Ax_{(i)}\, e^{-\theta xi} \left(1 - Ax_{(i)}\right)^{\alpha-1}\right)$$

$$= \alpha^{s-r+1} (\lambda\theta)^{s-r+1} k^{-\alpha(s-r+1)} \prod_{i=r}^{s} Ax_i\;\; e^{-\theta \sum_{i=r}^{s} x_i} \prod_{i=r}^{s} (1 - Ax_i)^{\alpha-1}$$

Let h=s-r+1, $W = (\lambda\theta)^h e^{\sum_{i=r}^{s} lnAx_{(i)}} e^{-\theta \sum_{i=r}^{s} x_{(i)}} e^{\sum_{i=r}^{s} ln\left(1 - Ax_{(i)}\right)^{-1}}$ and $\emptyset(x) = \sum_{i=r}^{s} ln(1 - Ax_i)^{-1}$,then we have

$$\prod_{i=r}^{s} f(x_i, \alpha) = W\alpha^h\, k^{-\alpha h} e^{-\alpha\emptyset(x)} \tag{8}$$

Continuing from equation (7), we find that:

$$(1-F(x_s))^{n-s}=\sum_{j=0}^{n-s}\binom{n-s}{j}(-1)^j(1)^{n-s-j}\left(F(x_s)\right)^j$$
$$=\sum_{j=0}^{n-s}\binom{n-s}{j}(-1)^j\,(1)^{n-s-j}\left(\left(\frac{1-Ax_s}{k}\right)^{\alpha}\right)^j$$
$$=\sum_{j=0}^{n-s}\binom{n-s}{j}(-1)^j\left(\frac{1-Ax_s}{k}\right)^{\alpha j}$$
$$(1-F(x_s))^{n-s}=\sum_{j=0}^{n-s}(-1)^j\binom{n-s}{j}e^{-\alpha j ln(1-Ax_s)^{-1}}e^{-\alpha j lnk} \quad (9)$$
$$(F(x_r))^{r-1}=\left(\left(\frac{1-Ax_r}{k}\right)^{\alpha}\right)^{r-1}=\left(\frac{1-Ax_r}{k}\right)^{\alpha(r-1)}$$
$$=(1-Ax_r)^{\alpha(r-1)}(k)^{-\alpha(r-1)}$$
$$(F(x_r))^{r-1}=e^{-\alpha(r-1)ln(1-Ax_r)^{-1}}e^{-\alpha(r-1)lnk} \quad (10)$$

Substitute equation (8),(9)and(10)in(7) we get

$$L(\alpha|x)=\left(\frac{n!\,W}{(r-1)!\,(n-s)!}\right)$$
$$\sum_{j=0}^{n-s}(-1)^j\binom{n-s}{j}\alpha^h e^{-\alpha[\emptyset(x)+hlnk+jln(1-Ax_s)^{-1}+jlnk+(r-1)ln(1-Ax_r)^{-1}+(r-1)lnk]}$$

Let $\mu_j(x)=\emptyset(x)+(h+j+r-1)lnk-jln(1-Ax_s)-(r-1)\ln(1-Ax_r)$

And W1=$\frac{n!W}{(r-1)!(n-s)!}$, Then $L(\alpha|x)=W1\sum_{j=0}^{n-s}(-1)^j\binom{n-s}{j}\alpha^h e^{-\alpha\mu_j(x)}$ (11)

3.2 Bayes procedure

By using the Bayes method to find the posterior function under gamma prior function which is given as follows:

$$\pi(\alpha)=\frac{b^a}{\Gamma a}\alpha^{a-1}e^{-\alpha b} \quad ,\alpha>0,b,a>0 \quad (12)$$

The posterior function can be found form the following relation:

$$P(\alpha|x)=\frac{L(\alpha|x)\,\pi(\alpha)}{\int_0^\infty L(\alpha|x)\ \pi(\alpha)d\alpha}$$

By using (11)and (12) the posterior function becomes:

$$P(\alpha|x)=\frac{W1\sum_{j=0}^{n-s}(-1)^j\binom{n-s}{j}\alpha^h e^{-\alpha\mu_j(x)}\frac{b^a}{\Gamma a}\alpha^{a-1}e^{-\alpha b}}{\int_0^\infty W1\sum_{j=0}^{n-s}(-1)^j\binom{n-s}{j}\alpha^h e^{-\alpha\mu_j(x)}\frac{b^a}{\Gamma a}\alpha^{a-1}e^{-\alpha b}d\alpha}$$
$$=\frac{W1\frac{b^a}{\Gamma a}\sum_{j=0}^{n-s}(-1)^j\binom{n-s}{j}\alpha^{a+h-1}e^{-\alpha(\mu_j(x)+b)}}{W1\frac{b^a}{\Gamma a}\sum_{j=0}^{n-s}(-1)^j\binom{n-s}{j}\int_0^\infty \alpha^{a+h-1}\,e^{-\alpha(\mu_j(x)+b)}d\alpha}$$

Using the relation ship $\int_0^\infty x^{\alpha-1}\,e^{-\beta x}dx=\frac{\Gamma\alpha}{\beta^\alpha}$ (13)

Then, we get

$$P(\alpha|x)=\frac{\sum_{j=0}^{n-s}(-1)^j\binom{n-s}{j}\alpha^{a+h-1}e^{-\alpha(\mu_j(x)+b)}}{\sum_{j=0}^{n-s}(-1)^j\binom{n-s}{j}\frac{\Gamma(a+h)}{\left(\mu_j(x)+b\right)^{(a+h)}}} \quad (14)$$

3.2.1 Quadratic loss function.

The Bayes estimator for α using the quadratic loss function is given as follows:[8]

$$\hat{\alpha}_Q = \frac{E(\alpha^{-1}|x)}{E(\alpha^{-2}|x)}$$

$$E(\alpha^{-1}|x) = \int_0^\infty \alpha^{-1} P(\alpha|x)\, d\alpha \quad (15)$$

By compensating (14) in equation (15) we can get

$$E(\alpha^{-1}|x) = \int_0^\infty \alpha^{-1} \frac{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \alpha^{h+a-1} e^{-\alpha(\mu_j(x)+b)}}{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(h+a)}{(\mu_j(x)+b)^{(h+a)}}}$$

$$= \frac{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \int_0^\infty \alpha^{h+a-2} e^{-\alpha(\mu_j(x)+b)} d\alpha}{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(h+a)}{(\mu_j(x)+b)^{(h+a)}}}$$

By equation (13) we get

$$E(\alpha^{-1}|x) = \frac{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(h+a-1)}{(\mu_j(x)+b)^{(h+a-1)}}}{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(h+a)}{(\mu_j(x)+b)^{(h+a)}}} \quad (16)$$

$$E(\alpha^{-2}|x) = \int_0^\infty \alpha^{-2} P(\alpha|x)\, d\alpha = \int_0^\infty \alpha^{-2} \frac{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \alpha^{h+a-1} e^{-\alpha(\mu_j(x)+b)}}{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(h+a)}{(\mu_j(x)+b)^{(h+a)}}} d\alpha$$

$$= \frac{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \int_0^\infty \alpha^{h+a-3} e^{-\alpha(\mu_j(x)+b)} d\alpha}{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(h+a)}{(\mu_j(x)+b)^{(h+a)}}}$$

$$E(\alpha^{-2}|x) = \frac{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(a+h-2)}{(\mu_j(x)+b)^{(a+h-2)}}}{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(a+h)}{(\mu_j(x)+b)^{(a+h)}}}$$

Then, the estimates will be as follows:

$$\hat{\alpha}_Q = \frac{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(h+a-1)}{(\mu_j(x)+b)^{(h+a-1)}}}{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(h+a)}{(\mu_j(x)+b)^{(a+h)}}} \cdot \frac{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(a+h)}{(\mu_j(x)+b)^{(a+h)}}}{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(a+h-2)}{(\mu_j(x)+b)^{(a+h-2)}}}$$

$$\hat{\alpha}_Q = \frac{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(h+a-1)}{(\mu_j(x)+b)^{(h+a-1)}}}{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(a+h-2)}{(\mu_j(x)+b)^{(a+h-2)}}}$$

$$\hat{\beta}_{Qi} = \frac{\sum_{j=0}^{m_i-s}(-1)^j \binom{m_i-s}{j} \frac{\Gamma(h+a-1)}{\left(\mu_j(y)+b\right)^{(h+a-1)}}}{\sum_{j=0}^{m_i-s}(-1)^j \binom{m_i-s}{j} \frac{\Gamma(a+h-2)}{\left(\mu_j(y)+b\right)^{(a+h-2)}}} \quad , i = 1,2,3$$

And the reliabilities estimation in equation (5)and (6), we get:

$$\hat{R}_{1Q} = \frac{\hat{\alpha}_Q}{\hat{\alpha}_Q + \hat{\beta}_Q} \qquad , \hat{R}_{2Q} = \frac{\hat{\alpha}_Q}{\hat{\alpha}_Q + \hat{\beta}_{1Q} + \hat{\beta}_{2Q} + \hat{\beta}_{3Q}}$$

3.2.2 Weighted loss function

The Bayes estimator for α, β using the weighted loss function which is given as follows:[8]

$$\hat{\alpha}_w = \frac{1}{E(\alpha^{-1}|x)} \tag{17}$$

By compensating (16) in (17), we get

$$\hat{\alpha}_w = 1 \Bigg/ \frac{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(h+a-1)}{\left(\mu_j(x)+b\right)^{(h+a-1)}}}{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(h+a)}{\left(\mu_j(x)+b\right)^{h+a}}}$$

$$\hat{\alpha}_w = \frac{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(h+a)}{\left(\mu_j(x)+b\right)^{(h+a)}}}{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(h+a-1)}{\left(\mu_j(x)+b\right)^{(h+a-1)}}}$$

$$\hat{\beta}_{wi} = \frac{\sum_{j=0}^{mi-s}(-1)^j \binom{mi-s}{j} \frac{\Gamma(h+a)}{\left(\mu_j(y)+b\right)^{(h+a)}}}{\sum_{j=0}^{mi-s}(-1)^j \binom{mi-s}{j} \frac{\Gamma(h+a-1)}{\left(\mu_j(y)+b\right)^{(h+a-1)}}} \quad , i = 1,2,3$$

And the reliabilities estimation in equation (5) and (6) we get:

$$\hat{R}_{1w} = \frac{\hat{\alpha}_w}{\hat{\alpha}_w + \hat{\beta}_w} \quad , \hat{R}_{2w} = \frac{\hat{\alpha}_w}{\hat{\alpha}_w + \hat{\beta}_{1w} + \hat{\beta}_{2w} + \hat{\beta}_{3w}}$$

3.2.3 Linear exponential loss function

The Bayes estimator for α, β using the linear exponential function is given as follows:[9]

$\hat{\alpha}_L = \frac{-1}{c} lnE(e^{-c\alpha}|x)\text{E}(e^{-c\alpha}|x)$

$$= \int_0^\infty e^{-c\alpha} P(\alpha|x)\, d\alpha = \int_0^\infty e^{-c\alpha} \frac{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \alpha^{h+a-1} e^{-\alpha(\mu_j(x)+b)}}{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(h+a)}{\left(\mu_j(x)+b\right)^{(h+a)}}} d\alpha$$

$$= \frac{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \int_0^\infty \alpha^{h+a-1} e^{-\alpha(\mu_j(x)+b+c)} d\alpha}{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(h+a)}{\left(\mu_j(x)+b\right)^{(h+a)}}}$$

$$E(e^{-c\alpha}|x) = \frac{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(h+a)}{\left(\mu_j(x)+b+c\right)^{(h+a)}}}{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(h+a)}{\left(\mu_j(x)+b\right)^{(h+a)}}} \tag{18}$$

$$\hat{\alpha}_L = \frac{-1}{c} ln\left(\frac{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(h+a)}{\left(\mu_j(x)+b+c\right)^{(h+a)}}}{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(h+a)}{\left(\mu_j(x)+b\right)^{(h+a)}}}\right)$$

$$\hat{\beta}_{iL} = \frac{-1}{c} ln\left(\frac{\sum_{j=0}^{mi-s}(-1)^j \binom{mi-s}{j} \frac{\Gamma(h+a)}{\left(\mu_j(y)+b+c\right)^{(h+a)}}}{\sum_{j=0}^{mi-s}(-1)^j \binom{mi-s}{j} \frac{\Gamma(h+a)}{\left(\mu_j(y)+b\right)^{(h+a)}}}\right) \quad , i = 1,2,3$$

And the reliabilities estimation in equation (5)and (6), we get:

$$\hat{R}_L = \frac{\hat{\alpha}_L}{\hat{\alpha}_L + \hat{\beta}_L} \quad , \hat{R}_{2L} = \frac{\hat{\alpha}_L}{\hat{\alpha}_L + \hat{\beta}_{1L} + \hat{\beta}_{2L} + \hat{\beta}_{3L}}$$

3.2.4 Non-Linear exponential loss function

The Bayes estimator for α, β using the non-linear exponential loss function which can be defined as follows:[10]

$$\hat{\alpha}_N = \frac{-1}{c+2}\left(lnE(e^{-c\alpha}|x) - 2E(\alpha|x)\right) \tag{19}$$

$$E(\alpha|x) = \int_0^\infty \alpha P(\alpha|x)d\alpha = \int_0^\infty \alpha \frac{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \alpha^{h+a-1} e^{-\alpha(\mu_j(x)+b)}}{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(h+a)}{\left(\mu_j(x)+b\right)^{h+a}}} d\alpha$$

$$E(\alpha|x) = \frac{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \int_0^\infty \alpha^{h+a}\, e^{-\alpha(\mu_j(x)+b)} d\alpha}{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(h+a)}{\left(\mu_j(x)+b\right)^{(h+a)}}}$$

$$E(\alpha|x) = \frac{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(h+a+1)}{\left(\mu_j(x)+b\right)^{(h+a+1)}}}{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(h+a)}{\left(\mu_j(x)+b\right)^{(h+a)}}} \tag{20}$$

By compensating (18) and (20) in equation (19)

$$\hat{\alpha}_N = \frac{-1}{c+2}\left[ln\left(\frac{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(h+a)}{\left(\mu_j(x)+b+c\right)^{(h+a)}}}{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(h+a)}{\left(\mu_j(x)+b\right)^{(h+a)}}}\right) - 2\left(\frac{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(h+a+1)}{\left(\mu_j(x)+b\right)^{(h+a+1)}}}{\sum_{j=0}^{n-s}(-1)^j \binom{n-s}{j} \frac{\Gamma(h+a)}{\left(\mu_j(x)+b\right)^{(h+a)}}}\right)\right]$$

$$\hat{\beta}_{iN} = \frac{-1}{c+2}\left[ln\left(\frac{\sum_{j=0}^{mi-s}(-1)^j \binom{mi-s}{j} \frac{\Gamma(h+a)}{\left(\mu_j(y)+b+c\right)^{(h+a)}}}{\sum_{j=0}^{mi-s}(-1)^j \binom{mi-s}{j} \frac{\Gamma(h+a)}{\left(\mu_j(y)+b\right)^{(h+a)}}} \right) - 2\left(\frac{\sum_{j=0}^{mi-s}(-1)^j \binom{mi-s}{j} \frac{\Gamma(h+a+1)}{\left(\mu_j(y)+b\right)^{(h+a+1)}}}{\sum_{j=0}^{mi-s}(-1)^j \binom{mi-s}{j} \frac{\Gamma(h+a)}{\left(\mu_j(y)+b\right)^{(h+a)}}} \right) \right] \quad ,i = 1,2,3$$

And the reliabilities estimation in equation (5) and (6) we get

$$\hat{R}_{1N} = \frac{\hat{\alpha}_N}{\hat{\alpha}_N + \hat{\beta}_N} \qquad ,\hat{R}_{2N} = \frac{\hat{\alpha}_N}{\hat{\alpha}_N + \hat{\beta}_{1N} + \hat{\beta}_{2N} + \hat{\beta}_{3N}}$$

4. Simulation Study

In this section, Monte Carlo simulation is performed to compare the performance of different estimations of R_1 andR_2. The simulation study has been carried out for four samples size $n = 15,30,50,100$. The values r=5,10,16,34 and s=9,20,30,50, respectively, and the values of c=1 ,the parameter values of the shape parameter α, the prior distribution a=2 and b=3 the results have been sufficiently replicated for L=1000 time for each experiment. The result presented some numerical experiment to compare the performance of the Bayes estimators under gamma prior distribution and four loss functions proposed in the previous sections. we have presented the simulation results using MATLAB(2013) program .A simulation results are conducted to examine and compare the performance of the estimates for parameter to the MSE. The estimator has the smallest value of MSE as well as doubly type II censored sample as it is shown in the last column in Tables 3- 8.

Table 1: the experiments for real R_1 value

Experiments	α	β	θ	λ	R_1
1	1.2	3	0.5	0.7	0.28571
2	1.2	0.6	0.5	0.7	0.66667
3	2	3	0.5	0.7	0.40000

Table 2: the experiments for real R_2 value

Experiments	α	β_1	β_2	β_3	θ	λ	R_2
1	2.5	1.5	2	3	1.2	1	0.27778
2	2.5	3	2	3	1.2	1	0.23809
3	2.5	1.5	2.5	3	1.2	1	0.26315

Table 3: R_1 estimatores performance for Experiments 1by MSE

(n.m)	BQ	BW	BL	BNL	BEST
(15.15)	0.00925	0.00927	0.01002	0.00952	BQ
(30.30)	0.00487	0.00488	0.00525	0.00500	BW
(15.30)	0.00295	0.00335	0.00404	0.00418	BQ
(30,15)	0.02087	0.01528	0.01346	0.01228	BNL
(50,50)	0.00269	0.00270	0.00287	0.00257	BQ
(50,15)	0.01300	0.00810	0.00665	0.00571	BNL
(50,30)	0.00211	0.00196	0.00194	0.00188	BNL
(15,50)	0.00300	0.00435	0.00589	0.00623	BQ
(30,50)	0.00675	0.00745	0.00825	0.00815	BQ
(100,100)	0.00143	0.00143	0.00149	0.00145	BQ,BW
(100,15)	0.00161	0.00241	0.00272	0.00342	BQ
(100,30)	0.00729	0.00803	0.00758	0.00832	BQ
(100,50)	0.00517	0.00532	0.00488	0.00526	BL
(15,100)	0.01364	0.02074	0.02588	0.02719	BQ
(30,100)	0.02970	0.03215	0.03379	0.03418	BQ
(50,100)	0.01879	0.01926	0.01985	0.01973	BQ

Table 4: R_1 estimatores performance for Experiments 2 by MSE

(n.m)	BQ	BW	BL	BNL	BEST
(15.15)	0.00705	0.00704	0.00701	0.00703	BL
(30.30)	0.00418	0.00418	0.00416	0.00417	BL
(15.30)	0.01197	0.00863	0.00710	0.00685	BNL
(30,15)	0.00511	0.00498	0.00523	0.00541	BW
(50,50)	0.00271	0.00271	0.00270	0.00270	BL,BNL
(50,15)	0.00459	0.00638	0.00838	0.00874	BQ
(50,30)	0.00644	0.00707	0.00762	0.00764	BQ
(15,50)	0.00548	0.00394	0.00349	0.00364	BL
(30,50)	0.00302	0.00314	0.00307	0.00321	BL
(100,100)	0.00146	0.00146	0.00145	0.00145	BL,BNL
(100,15)	0.02201	0.03112	0.03804	0.03929	BQ
(100,30)	0.03331	0.03589	0.03767	0.03803	BQ
(100,50)	0.01902	0.01948	0.01986	0.01988	BQ
(15,100)	0.00770	0.01142	0.01347	0.01446	BQ
(30,100)	0.01554	0.01667	0.01683	0.01740	BQ
(50,100)	0.00994	0.01017	0.00997	0.01024	BQ

Table 5: R_1 estimatores performance for Experiments 3 by MSE

(n.m)	BQ	BW	BL	BNL	BEST
(15.15)	0.00574	0.00573	0.00557	0.00567	BW
(30.30)	0.00351	0.00350	0.00544	0.00348	BNL
(15.30)	0.00534	0.00385	0.00304	0.00324	BL
(30,15)	0.01830	0.01309	0.01094	0.01013	BNL
(50,50)	0.00262	0.00262	0.00259	0.00261	BL
(50,15)	0.01285	0.00786	0.00607	0.00538	BNL
(50,30)	0.00251	0.00249	0.00231	0.00245	BL
(15,50)	0.00379	0.00301	0.00297	0.00331	BL
(30,50)	0.00484	0.00538	0.00562	0.00581	BQ
(100,100)	0.00148	0.00148	0.00146	0.00147	BL
(100,15)	0.00252	0.00433	0.00508	0.00624	BQ
(100,30)	0.01224	0.01348	0.01297	0.01406	BQ
(100,50)	0.00871	0.00896	0.00842	0.00893	BL
(15,100)	0.00833	0.01375	0.01683	0.01858	BQ
(30,100)	0.02554	0.02776	0.02827	0.02927	BQ
(50,100)	0.01754	0.01798	0.01799	0.01825	BQ

Table 6:R_2 estimatores performance for Experiments 1by MSE

(n.m1,m2,m3)	BQ	BW	BL	BNL	BEST
(15.15,15,15)	0.00165	0.00164	0.00146	0.00158	BL
(30.30,30,30)	0.00149	0.00149	0.00137	0.00145	BL
(15.30,30,30)	0.00587	0.00401	0.00310	0.00294	BNL
(30,15,15,15)	0.00667	000410	0.00286	0.00278	BNL
(50,50,50,50)	0.00109	0.00109	0.00102	0.00106	BL
(50,15,15,15)	0.00349	0.00169	0.00117	0.00116	BNL
(50,30,30,30)	0.00156	0.00174	0.00176	0.00187	BQ
(15,50,50,50)	0.00372	0.00214	0.00156	0.00152	BNL
(30,50,50,50)	0.00164	0.00184	0.00176	0.00195	BQ
(100,100,100,100)	7.546e-04	7.542e-04	7.292e-04	7.456e-04	BL
(100,15,15,15)	0.00274	0.00503	0.00620	0.00698	BQ
(100,30,30,30)	0.01007	0.01091	0.01093	0.01141	BQ
(100,50,50,50)	0.00753	0.00771	0.00757	0.00777	BQ
(15,100,100,100)	0.00307	0.00605	0.00747	0.00891	BQ
(30,100,100,100)	0.01562	0.01732	0.01703	0.01827	BQ
(50,100,100.100)	0.01064	0.01096	0.01054	0.01102	BL

Table 7: R_2 estimatores performance for Experiments 2 by MSE

(n.m1,m2,m3)	BQ	BW	BL	BNL	BEST
(15.15,15,15)	0.00144	0.00143	0.00127	0.00137	BL
(30.30,30,30)	0.00117	0.00117	0.00108	0.00114	BL
(15.30,30,30)	0.00370	0.00244	0.00179	0.00175	BNL
(30,15,15,15)	0.00733	0.00455	0.00334	0.00309	BNL
(50,50,50,50)	9.060e-04	9.055e-04	8.555e-04	8.886e-04	BL
(50,15,15,15)	0.00458	0.00228	0.00153	0.00135	BNL
(50,30,30,30)	0.00097	0.00106	0.00103	0.00112	BQ
(15,50,50,50)	0.00248	0.00147	0.00112	0.00117	BL
(30,50,50,50)	0.00176	0.00200	0.00197	0.00215	BQ
(100,100,100,100)	5.671e-04	5.669e-04	5.488e-04	5.607e-04	BL
(100,15,15,15)	0.00133	0.00273	0.00339	0.00399	BQ
(100,30,30,30)	0.00691	0.00753	0.00742	0.00786	BQ
(100,50,50,50)	0.00523	0.00537	0.00518	0.00539	BL
(15,100,100,100)	0.00273	0.00542	0.00679	0.00803	BQ
(30,100,100,100)	0.01456	0.01615	0.01601	0.01708	BQ
(50,100,100.100)	0.01009	0.01039	0.01010	0.01048	BQ

Table 8: R_2 estimatores performance for Experiments 3 by MSE

(n.m1,m2,m3)	BQ	BW	BL	BNL	BEST
(15.15,15,15)	0.00160	0.00159	0.00140	0.00153	BL
(30.30,30,30)	0.00127	0.00127	0.00117	0.00124	BL
(15.30,30,30)	0.00485	0.00325	0.00246	0.00235	BNL
(30,15,15,15)	0.00700	0.00432	0.00306	0.00292	BNL
(50,50,50,50)	0.00103	0.00103	0.00096	0.00100	BL
(50,15,15,15)	0.00363	0.00173	0.00117	0.00111	BNL
(50,30,30,30)	0.00140	0.00154	0.00152	0.00163	BQ
(15,50,50,50)	0.00347	0.00208	0.00155	0.00154	BNL
(30,50,50,50)	0.00175	0.00196	0.00190	0.00209	BQ
(100,100,100,100)	6.210e-04	6.207e-04	5.993e-04	6.134e-04	BL
(100,15,15,15)	0.00211	0.00403	0.00499	0.00570	BQ
(100,30,30,30)	0.00889	0.00965	0.00961	0.01009	BQ
(100,50,50,50)	0.00639	0.00655	0.00640	0.00660	BQ
(15,100,100,100)	0.00299	0.00590	0.00732	0.00870	BQ
(30,100,100,100)	0.01562	0.01732	0.01709	0.01829	BQ
(50,100,100.100)	0.00990	0.01020	0.00986	0.01027	BL

5. Conclusion

In Tables 3-8, we observed that the value of MSE is decreases by increasing the sample size n, m and n,m1,m2,m3 for doubly type II censored and Bayes estimators . In general, we also observed that the best performances were BQ,BL,BNL and BW under the Bayes method.

Acknowledgments

The authors would like to thank Mustansiriyah University in Baghdad-Iraq for its support in the present work.

References

[1] K. Adamidis and S. Loukas S, "A lifetime distribution with decreasing failure rate", *Statistics And probability Letters*, vol. 39, pp. 35-42, 1998.

[2] C. Kus , "A new lifetime distribution", *Computational Statistics and Data Analysis*, vol. 51, no. 9, pp. 4497-4509, 2007.

[3] W. Barreto-Souza and F. Cribari-Neto, "A generalization of the Exponential-poison distribution", *Statistics and probability Letters*, vol. 79, no. 24, pp. 2493-2500, 2009.

[4] A. Baklizi, (2014). "Bayesian inference for Pr (Y< X) in the exponential distribution based on records", *Applied Mathematical Modelling,* vol .38, no. (5-6), pp. 1698-1709, 2014.

[5] B. Saraço˘glu , and M. F. Kaya, " Maximum Likelihood Estimation and Confidence Intervals of System Reliability for Gompertz Distribution in Stress-Strength Models", *Selçuk Journal of Applied Mathematics*, vol.8, no. 2, pp.25-36, 2007.

[6] N. S. Karam, and H. G. R. Ramk, "Reliability Estimation for a Component Exposed Tow, Three Independent Stresses Based on Weibull and Inverse Lindly Distribution", *Mathematical Theory and Modeling*, vol. 6, no. 8, pp. 2225-0522, 2016.

[7] A. C. Cohen, "Maximum Likelihood Estimation in the Weibull Distribution Based on Complete and on Censored Samples" *American Statistical Association, Technometrics*, vol. 7, no. 4, pp. 579-588, 1965.

[8] A. R. Mezaal,., Mansour, & N. S. Karm, "Multicomponent stress-strength system reliability estimation for generalized exponential-poison distribution". In *Journal of Physics: Conference Series* (vol. 1591, no. 1, p. 012042), 2020. IOP Publishing. 012042. IOP Publishing.

[9] S. Islam, "Loss functions, Utility functions and Bayesian Sample Size Determination" PH.D. Thesis, University of London.26, 2011.

[10] H. Rahman, M. K. Roy and A.R. Baizid, "Bayes Estimation under Conjugate Prior for the Case of Power Function Distribution" *American Journal of Mathematics and Statistics*, vol. 2, no. 3, pp.44-48, 2012.